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INVESTMENT AND INCOME DISTRIBUTION PATTERN UNDER MUSHARKA FINANCE: The Certainty Case

Shamim Ahmad SIDDIQUI
and Asad ZAMAN*

An attempt has been made to provide microeconomic foundation for a macroeconomic system based on Islamic financial principles. Using a Ricardo type corn model in a deterministic framework, some aspects of investment and income distribution pattern under a financing system similar to Musharka finance are analyzed and compared to the traditional fixed interest system. It is shown that without any element of uncertainity, the interest system, unlike the equity system, is Pareto optimal. Implications of the results for Islamic banking are discussed, and mode of government intervention is suggested which can make the equity finance system more efficient.

Several attempts have been made to analyze the basic characteristics of a profit and loss sharing economy based on Islamic financial principles. All these studies, which are helpful in understanding the different aspects of the new system, have been done in a static framework with certainty. Anwar (1987) used Sargent's macroeconomic model (1979) and replaces the rate of interest by the 'rate of return' on capital. His main conclusion was that in the sharing system, households are

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insured against unexpected inflation. In other words, as the rate of inflation increases, so does the total revenue of firms (in nominal terms). Banks and households, by sharing the firm's profit, get a higher rate of return. Chaudhry (1986) found Khan's consumption function appropriate for his model. Presenting the investment function for the new system, he derived the IS and LM curves which can have negative or a positive slopes. He does not, however, discuss the role of the government in the Islamic system and it's affect on the two curves. Khan (1986) used Fernandez's (1984) model, which is an improved version of Metzler (1951), to study the comparative stability of the Islamic financial system. An evaluation of these models reveals that they lack a sound micro foundation. The primary goal of this paper is to provided such a foundation with a view to build a comprehensive macroeconomic model in the future.[1] We analyze investment and income distribution patterns under a sharing system similar to Musharka finance in a deterministic framework, and compare it to the fixed interest case.

Principle of Musharka Finance

Prohibition of interest paid on loans and investment is a fundamental axiom of Islamic finance.[2] To replace the institution of interest, muslim scholars and jurists have come up with many financial techniques which are in conformity with the Shariah.[3] One of the preferred techniques in Islamic finance is "Musharka" This is a form of business arrangement in which a number of partners pool financial resources to undertake a commercial/industrial enterprise. All parties can invest in equal or variable proportions, and profits and losses are shared strictly in relation to their respective contributions.[4] Musharka financing closely corresponds to an equity market, in which shares can be acquired by the public, commercial banks, the central bank and the government. For example, firms attempting to raise funds for investment can use this mechanism by offering Musharka certificates in the market. Such certificates would in effect, be transferable corporate instruments secured by the assets of the company. Their prices and the implicit rate of return would be determined by the market.[5]

[1] Two other papers are also devoted for this purpose: Siddiqui and Zaman [The Uncertainty Case (1989)]; and Siddiqui, "Saving, investment and income distribution pattern under Mudarha finance," Review of Islamic Economics, (forthcoming).

[2] Several arguments have been provided by Islamic scholars to explain and justify their positions. The arguments against charging a fixed interest rate has placed major emphasis on the alleged lack of a satisfactory theory of interest in the western economic tradition. Some useful discussion on this issue can be found in Iqbal and Mirakhor (1987), Khan and Mirakhor (1986), Khan (1986), and Siddiqui (1989) and the references cited therein.

[3] See, Iqbal and Mirakhor, (1987).

[4] See, Khan and Mirakhor, (1986).

[5] Ibid.

The Model

In order to keep the model manageable and concentrate on a single issue, we have made many simplifying assumptions. Specially, we have worked with only one good and one factor of production, capital. This is because our main concern is to explore the behavioral and distributional characteristics of Musharka finance. In following the Ricardian pattern, we consider a primitive society of an individuals producing and consuming a single good, corn. Each individual has a piece of land which is identical in size and quality. All of them have identical utility and production functions. Each individual lives for two periods. In the first period he gets Y_i, output of the corn planted by his elders, which may be different for different individuals. In the first period, there exists a perfect market for borrowing and lending in the interest system. In the sharing case too, one can lend or borrow.[6] However, the mode of transaction is similar to the principle of Musharka finance. In Musharka finance individuals pool their resources and lend them to the producers/entrepreneurs. Whether one (or more) of the producer is from amongst the lenders is immaterial for the analysis of Musharka finance, if one gets a separate reward for the labour input. In our model both types of farmers, lenders and borrowers, produce and consume a single commodity, corn. Profits of the borrower are shared by them, the lenders share of the profit is not in proportion to his contribution. His share is less than that because the borrower gets a fraction of the profit as remuneration for his managerial input which is determined by the market.

Lenders and borrowers agree to share the profit according to the pre-specified terms and conditions described below.[7] In the first period, therefore, an individual has to make a decision about the amount of borrowing and planting. In the second period, he has to pay back the loan and the interest in the traditional case, and share the profit (instead of interest) in the sharing case. For simplicity, but without an essential loss of generality, we assume that no one in the second period does any planting for the next generation. Given the utility and the production functions, each individual maximizes his life time (two period) utility subject to the budget constraint.

The order of the paper is as follows: In Section I we present the fixed interest model, whereas Section II deals with the sharing case. The comparative static analysis of the sharing model is done in Section III. Numerical solutions of the model are presented and analyzed in Section IV, while some broad conclusions are contained in Section V.

I. The Fixed Interest Case

The utility maximization problem for the ith farmer can be written as:

[6] Throughout this paper we will be using 'sharing' for the Musharka finance.

[7] In the present case of certainty, the question of loss sharing does not arise.

$$\text{Max } U_i(C_{1i}) + b\, U_i(C_{2i}), \qquad 0 < b < 1 \tag{1.1}$$

subject to $$C_{1i} = Y_{1i} + B_i - P_i$$

and $$C_{2i} = f(P_i) - (1+r)B_i$$

where b is the discount factor for consumption in the second period, B_i is borrowing or lending in the first period; r is the rate of interest, endogenously determined by the system, and $f(P_i)$ is the output of corn in the second period due to P_i amount of corn planted in the first period. Taking P_i and B_i as the choice variables, the first order conditions (in the implicit function form) can be written as:

$$U'(Y_{1i}+B_i-P_i) = b\, U'[f(P_i) - (1+r)B_i][f'(P_i)] \tag{1.2}$$

$$U'(Y_{1i}+B_i-P_i) = b\, U'[f(P_i) - (1+r)B_i](1+r)$$

or $$U'(C_{1i}) = b f'(P_i)\, U'(C_{2i}) \tag{1.2a}$$

$$f'(P_i) = 1+r \tag{1.2b}$$

Equation (1.2a) implies that the ratio of marginal utilities of period one to that of period two must be equal to the discount rate times the marginal product. The second equation is also a standard result: at the optimal planting level, marginal product must be equal to 1+r. This is true for any utility and production function. We can derive explicit expressions for optimal levels of P_i, B_i and the equilibrium level of the rate of interest by using the following log utility and power production function; both having desirable properties of positive first derivatives and negative second derivative.[8]

$$U(C_{ti}) = Ln(C_{ti}) \qquad t=1, 2; \quad i=1,2,\ldots,n. \tag{1.3}$$

$$f(P_i) = AP_i^k \qquad A > 0; \quad 0 < k < 1. \tag{1.4}$$

Now we can write the maximization problem in the explicit form:

$$\underset{P_i, B_i}{\text{Max}}\; Ln(Y_{1i}+B_i-P_i) + b\, Ln[AP_i^k-(1+r)B_i].$$

[8] The log utility function exhibits diminishing marginal utility and risk averse behavior, whereas the power production function implies diminishing marginal productivity. The choice of these particular utility and production functions, has made the numerical results much more meaningful, as we will see later.

First Order Conditions with respect to P_i and B_i can be written as:

$$\frac{1}{(Y_{1i}+B_i-P_i)} = \frac{bkAP_i^{k-1}}{[AP_i^k-(1+r)B_i]} \tag{1.5}$$

$$\frac{1}{(Y_{1i}+B_i-P_i)} = \frac{b(1+r)}{[AP_i^k-(1+r)B_i]} \tag{1.6}$$

Dividing (1.5) by (1.6) gives

$$kAP_i^{k-1} = 1+r \quad \text{or} \quad P_i = \left[\frac{kA}{1+r}\right]^{1/(1-k)} \tag{1.7}$$

Cross multiplication of equation (1.6) implies

$$(1+r)(1+b)B_i = AP^k_i - b(1+r)(Y_{1i}-P_i)$$

or

$$Bi = \frac{AP^k_i - b(1+r)(Y_{1i}-P_i)}{(1+r)(1+b)} \tag{1.8}$$

where $P_i = [kA/(1+r)]^{1/(1-k)}$

In equilibrium, total borrowing must equal to lending or

$$\sum_i B_i = 0 = \sum_i \left[\frac{AP_i^k - b(1+r)(Y_{1i}-P_i)}{(1+r)(1+b)}\right] \tag{1.9}$$

Simplifying (1.9) and using the value of P_i as given in equation (1.7) will give the equilibrium value of r:

$$r^* = \left[\frac{An(Ak)^{(k/1-k)} + bn(Ak)^{(1/1-k)}}{[b \sum_i Y_{1i}]^{1-k}}\right]^{1-k} - 1 \tag{1.10}$$

where n is the total number of farmers (borrowers and lenders).

II. The Musharka or Sharing Case

In the sharing model, maximization problems are different for the borrower and the lender. The maximization problem for the ith borrower can be written as:

$$\text{Max } U(C_{1i}) + b\, U(C_{2i})$$

$$\text{s.t. } C_{2i} = Y_{1i} + B_i - P_i \tag{2.1}$$

and

$$C_{2i} = f(P_i) - m(B_i/P_i)[f(P_i) - P_i] - B_i$$

Here mB_i/P_i is the share of profit going to the lender, the value of m being determined by the stystem. This arrangement is similar to the famous Islamic mode of finance known as Musharka.[9] Again taking P_i and B_i as choice variable, first order conditions (in implicit form) for this system can be written as:

$$U_i'(C_{1i}) = b\, U_i'(C_{2i})\Big[f'(P_i) - mB_i \Big\{\frac{f'(P_i)}{P_i} - \frac{f(P_i)}{P_i^2}\Big\}\Big] \tag{2.2}$$

$$U_i'(C_{1i}) = b\, U_i'(C_{2i})\Big[\frac{m}{P_i}\Big\{f(P_i) - P_i\Big\} + 1\Big] \tag{2.3}$$

Dividing (2.2) by (2.3) implies, after simplification

$$mf(P_i) - mP_i + P_i = P_i f'(P_i) + mB_i f(P_i) / P_i - mf'(P_i)B_i,$$

or

$$B_i = \frac{P_i^2\{1-m-f'(P_i)\}+mP_i f'(P_i)}{m[f(P_i) - f'(P_i)P_i]} \tag{2.4}$$

A solution to (B_i,P_i) may exist (as we will see in a moment that the Jacobian is non-zero), but no explicit solution could be found in the general case using equations (2.3) and (2.4). However assuming $U(C_{ti}) = Ln(C_{ti})$ and $f(P_i) = AP_i^k$, equation (2.3) can also be used to express B_i as a function of P_i.

or

$$B_i = \frac{AP_i^k - b(Y_{1i} - P_i)\,[mAP_i^{k-1} - m+1]}{(1+b)\,[mAP_i^{k-1} - m+1]} \tag{2.5}$$

We use different combinations of appropriate utility and production functions,

[9] In this mode of transaction any number of people can participate in forming a fund for investment. The rate of profit, for an individual depends on his/her contribution to the fund. For a detailed discussion, see Khan and Mirakhor (1986).

hoping that some of them may result in first order conditions simple enough to get analytical solutions for the choice variables (P_i and B_i). Unfortunately, we did not succeed even though we used special values for some of the key parameters. The only choice left was to use numerical techniques to get solutions for the system. We use equations (2.4) and (2.5) to get numerical solutions for B_i and P_i. These two equations are shown graphically in Figure 1, using specific values of the parameters and exogenous variables for power production and log utility functions. The intersection of the two curves gives all possible solutions for the appropriate range of the choice variables. To check that the solution maximizes utility, we first verify that the first order condition holds and then use the objective function to verify the second order condition. In some cases we got only one solution for P_i and B_i for a given value of m. If we get more than one solution, usually only one of them generates the highest utility level. However, even if we have more than one solution generating the same level of utility, only one of them is finally acceptable. This is because for a given m and P_i, a unique amount of loan will be offered by the lender.[10] We continue changing the value of m until we arrive at the value $B_i = B_j$.[11]

Lenders form an investment cooperative which collects all the loanable corn and gives loans to different farmers. The total profit of the cooperative is given as:

$$T.P. = \sum_i (B_i/P_i)(AP_i^k - P_i) \qquad (2.6)$$

The actual rate of return to the cooperative or to the (lenders), if we assume that the cooperative works at no cost, is:

$$ra = T.P./\sum_i B_i \qquad (2.7)$$

The actual return of the jth lender would be $(1+ra)B_j$.

The maximization problem to the jth lender can now be defined as:

$$\text{Max } U_j(C_{1j}) + b\,U_j(C_{2j}), \qquad 0 < b < 1$$

Subject to $$C_{1j} = Y_{1j} - B_j - P_j \qquad (2.8)$$

and $$C_{2j} = f(P_j) + (1+r_e)B_j$$

Taking P_j and B_j as the choice variables, the first order conditions (in the implicit functional form) can be written as:

[10] Lender's problem is defined and solved below.

[11] Expression for B_j is given in equation (2.10).

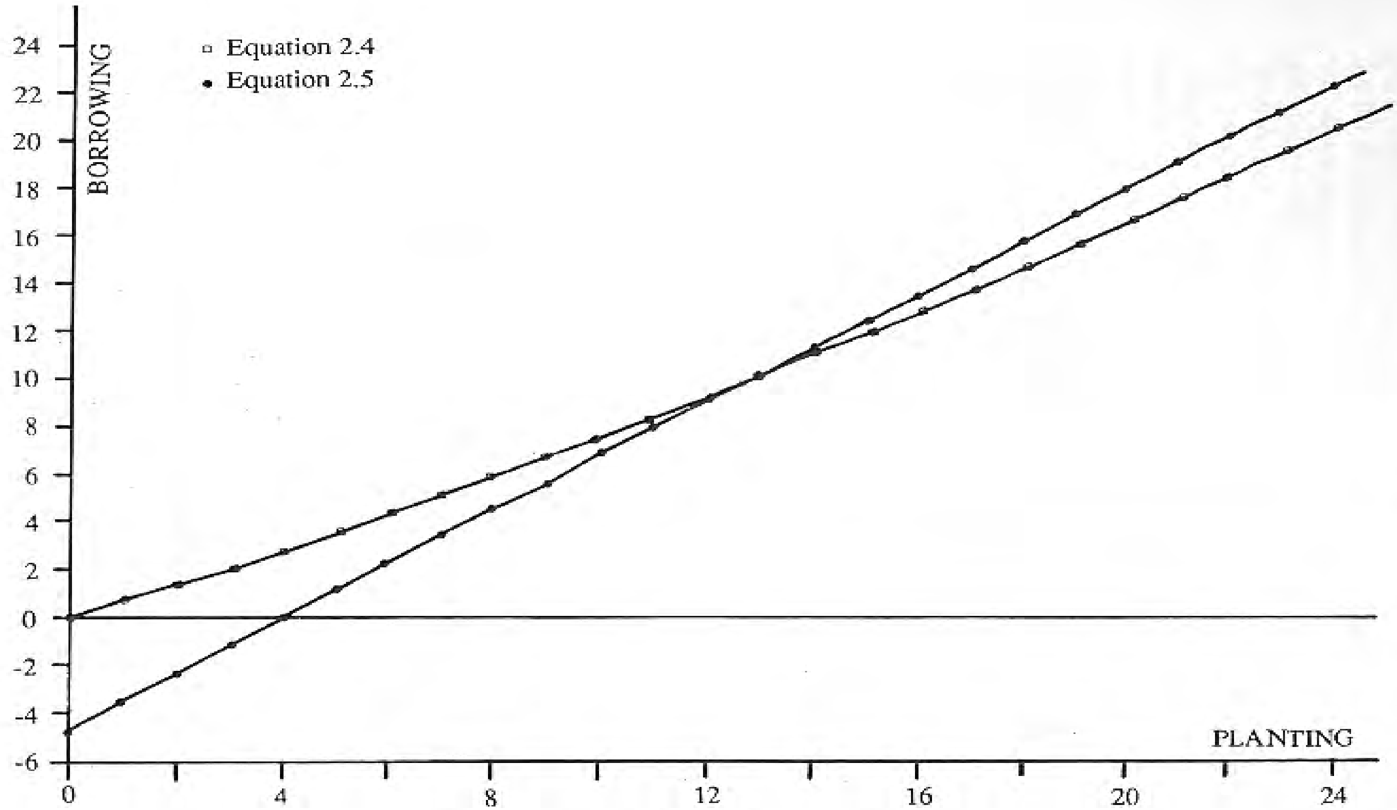


FIGURE 1

Certainty: Sharing; Borrower's Case

$A = 14.8$, $m = 0.7$, $k = 0.5$, $d = 0.9$

$$U'(Y_{1j}-B_j-P_j) = b\,U'[f(P_j)+(1+r_e)B_j]\,\{f'(P_j)\} \tag{2.9a}$$

$$U'(Y_{1j}-B_j-P_j) = b\,U'[f(P_j)+(1+r_e)B_j](1+r_e) \tag{2.9b}$$

(a) $$U'(C_{1j}) = b\,f'(P_j)\,U'(C_{2j})$$

(b) $$f'(P_j) = 1+r_e.$$

These conditions are similar to the interest rate case and could be interpreted accordingly. Using the log utility and power production function, the lender's problem can be written as follows:

$$\max_{P_j,B_j} Ln(Y_{1j}-B_j-P_j) + bLn[(AP_j^k+(1+re)B_j]. \tag{2.10}$$

where $$Y_{1j}-B_j-P_j = C_{1j} \text{ and } AP_j^k+(1+r_e)B_j = C_{2j}$$

where r_e is the expected rate of return for each farmer. In equilibrium r_e = ra, and $\sum_i B_i = \sum_j B_j$.

First Order Conditions with respect to P_j and B_j are:

$$\frac{1}{(Y_{1j}-B_j-P_j)} = \frac{bkAP_j^{k-1}}{[AP_j^k+(1+ra)B_j]} \tag{2.11}$$

$$\frac{1}{(Y_{1j}-B_j-P_j)} = \frac{b(1+ra)}{[AP_j^k+(1+ra)B_j]} \tag{2.12}$$

Dividing (2.11) by (2.12) gives:

$$KAP_j^{k-1} = 1+ra$$

or $$P_j = \left[\frac{KA}{1+ra}\right]^{1/(1-k)} \tag{2.13}$$

Cross multiplication equation (2.10) implies

$$(1+ra)(b-1)B_j = -AP_j^k+b(1+ra)(Y_{1j}-P_j)$$

or $$B_j = \frac{AP_j^k-b(1+ra)Y_{1j}-P_j)}{(1+ra)(1-b)} \tag{2.14}$$

This equation is used with numerical analysis to find the equilibrium values of lending and borrowing.

III. Some Interpretation of the First Order Conditions and Comparative Static Analysis of the Sharing Model

We consider the first order conditions for the borrower i.e., (2.2) and (2.3).

$$U_i'(C_{1i}) = b\, U_i'(C_{2i})\left[f'(P_i) - mB_i\left\{\frac{f'(P_i)}{P_i} - \frac{f(P_i)}{P_i^2}\right\}\right] \tag{2.2}$$

$$U_i'(C_{1i}) = b\, U_i'(C_{2i})\left[\frac{m}{P_i}\left\{f(P_i) - P_i\right\} + 1\right] \tag{2.3}$$

Dividing (2.2) by (2.3) and simplyfying

$$f'(P_i)(1-mB_i/P_i)+[mf(P_i)/P_i](B_i/P_i-1) = 1-m$$

or

$$f'(P_i) = f'(P_i)mB_i/P_i+mf(P_i)/P_i-mB_if(P_i)/P_i*P_i) + 1 \tag{3.1}$$

On the other hand, the two first order conditions for the representative lender are

$$U'(Y_{ij}-B_j-P_j) = b\, U'[f(P_j)+(1+r)B_j][f'(P_j)] \tag{2.9a}$$

$$U'(Y_{ij}-B_j-P_j) = b\, U'[f(P_j)+(1+r)B_j](1+r) \tag{2.9b}$$

Dividing (2.9a) by (2.9b), and then simplifying, we have

$$f'(P_j) = 1+ra = 1+m[f(P_i)-P_i]/P_i. \tag{3.2}$$

For $f'(P_i)$ and $f'(P_j)$ to be equal it must be true that

$$mB_i/P_i[f(P_i)/P_i-f'(P_i)] = 0 \tag{3.3}$$

The two terms in parenthesis in equation (3.3) are the average and the marginal products respectively. This in turn implies that the marginal product (and hence the function) for the lender and the borrower will not be equal unless we assume constant returns to scale. For the power production function, AP is always greater than MP which means the borrower will always plant more than the lender.

In order to check that we will always get the interior solution for $0 < m < 1$, we would like to look at the amount of borrowing and lending when m approaches zero or one.

For m = 0, the first order condition for the borrower i.e., (3.1) becomes

$$f'(P_i = 1).$$

This implies that the borrower will plant up to the point where the marginal product becomes one. This makes sense as he keeps all the profit and pays back only the principal. For the lender the value of ra[$= m\{f(P_i) - P_i\}/P_i$] becomes zero, which does not give him any incentive to lend. Consequently, the market does not clear in this situation.

For m = 1, equation (3.1) becomes

$$(B_i / P_i - 1)[f(P_i)/P_i - f'(P_i)] = 0;$$

which implies that either AP = MP, or $B_i = P_i$. For a non-constant returns to scale production function this means that the borrower will not plant more or less than the amount borrowed. The reason is obvious: he has to pay back the principal as well as all the profit. Consequently, he will not have any incentive to borrow. For the lender, with m = 1, ra becomes much greater, meaning greater supply while the demand for loans is zero. Again the market does not clear.

We will now consider the second order conditions to do the comparative static analysis.

The second order conditions for the borrower are as follows:

$$U''(C_{1i})+dU'(C_{2i})+Z+X^2\, dU''(C_{2i}) = F_i^1 \quad (3.4)$$

$$U''(C_{1i})+d\, M^2\, U''(C_{2i}) = F_{22}(\cdot) \quad (3.5)$$

$$- U''(C_{1i})+d\, U'(C_{2i})\, N+d\, XMU''(C_{2i}) = F_{12}(\cdot) = F_{21}(\cdot) \quad (3.6)$$

where

$Z = f''(P_i) - mB_i f''(P_i)/P_i+2mB_i f'(P_i)/(P_i^2)-2mB_i f(P_i)/(P_i^2)$;

$X = f''(P_i) - mB_i f'(P_i)/P_i+mB_i f(P_i)/(P_i^2)$;

$N = mf(P_i)/P_i^2$, and

$M = m - 1 - mf(P_i)P_i$.

The relevant Hessian matrix for the first order conditions is:

$$F_1 = 0 \text{ and } F_2 = 0.$$

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

To have a unique and stable solution F_{11} and F_{22} should be negative and the determinant of the above matrix must be positive.

Assuming that $U''(C_{1i}) = a$, $U'(C_{2i}) = u$ and $U''(C_{2i}) = h$,

$$F_{11}*F_{22}-F_{12}*F_{21} = a+ahM^2+abuZ+duhZM^2+abhX^2+ b\,h^2X^2\,M^2 - [-a+buN+bhXM]^2 \tag{3.7}$$

For a utility function with decreasing but positive marginal utility and a production function with a positive first derivative and a negative second derivative, we can determine the sign of the following variables:

$$a < 0 : u > 0 ; h < 0 : x > 0 : M < 0 ; N > 0 :$$

if AP > MP e.g., power production function.

$$Z < 0 : \text{if } AP > MP \tag{3.8}$$

This confirms that the sign of F_{11} and F_{22} is negative (implying that $F_{11} * F_{22} > 0$). However, as F_{21} (= F_{12}) is positive the sign of the determinant will depend on the magnitudes of $F_{11} * F_{22}$ and $F_{21} * F_{12}$ which are unknown in general and can be found only if we know the values of all the parameters. Even if we assume that the determinant is indeed strictly positive, it is not enough to get unambiguous comparative static results. For example, suppose we want to know the sign of dP_i/dA

$$\frac{dP_i}{dA} = \frac{\begin{vmatrix} -dF_1/dA & F_{12} \\ -dF_2/dA & F_{22} \end{vmatrix}}{|\,J\,|} \tag{3.9}$$

We already know that F_{12} is positive and F_{22} is negative. For a power production function and log utility function, first order condition for the borrower with respect to P_i and B_i can be written as:

$$\frac{-1}{Y_{1i}+B_i-P_i} + \frac{[kAP_i^{k-1} - mA(k-1)B_iP_i^{k-2}]}{AP_i^k - mB_i/P_i\{AP_i^k-P_i\} - B_i} = 0 \qquad (3.10a)$$

$$\frac{-1}{Y_{1i}+B_i-P_i} + \frac{b[-m/P_i\{AP_i^k - P_i\} - B_i}{AP_i^k - mB_i/P_i\{AP_i^k-P_i\} - B_i} = 0 \qquad (3.10b)$$

$$dF_i/dA = 1/V_i^2[V\{bkP_i^{k-1} - bm(k-1)B_iP_i^{k-2}] - W_1\{P_i^k - mB_iP_i^k/P_i\}] \qquad (3.11)$$

$$dF_2/dA = 1/V^2[V\{-bmP_i^k/P_i\} - W_2\{P_i^k-mB_iP_i^k/P_i\}] \qquad (3.12)$$

where $V = AP_i^k - mB_i/P_i\{AP_i^k-P_i\} - B_i$,
$W_1 = b[kAP_i^{k-1}-mA(k-1)B_iP_i^{k-2}]$ and
$W_2 = b[-m/P_i\{AP_i^k-P_i\} - B_i]$.

V is nothing but consumption for the second period and hence positive. W_1 is also positive as k-1 is less than zero. W_2 is negative because both terms in the square bracket are negative. As both terms in the square bracket of equation (3.11) are positive, the sign of dF_i/dA is ambiguous. Similarly, both the terms in the square bracket in equation (3.12) are negative making the sign of dF_2/dA unclear. Consequently, it is difficult to determine the sign of dP_i/dA and dB_i/dA. The same is true for the other parameters k and Y_i.

IV. Numerical Solution of the Certainty Model and their Interpretation

As our comparative static analysis could not establish signs for the different derivatives clearly, we decided to do a numerical exercise. This would not only establish both signs for the derivations but also check the robustness of the model with respect to different values of the parameters. Furthermore, this exercise enables us to highlight and quantify the differences in the two types of economies.

In the numerical analysis one has to choose suitable and appropriate values of the different parameters and exogeneous variables. As our main concern is in exploring the saving and investment behaviour of the borrower and the lender in the two systems, we made certain that for given values of endowments the values of the parameters are such that:

(a) they satisfy the conditions imposed by theory, i.e., the value of A, which can be interpreted as the state of technology, has to be greater than zero. Similarly, the value of k has to be less than one but greater than zero, implying a decreasing returns to scale technology. Finally, the discount rate b has to be a positive fraction.

(b) a positive amount is borrowed and lent. We first look at Table 1 where we have solutions of the model for A = B, k = 0.5 and b = 1.0. (To check the robustness of our results, we have solved the model numerically for different values of the parameters. The results are graphically shown in the Appendix). For simplicity, we have considered only two farmers: one lender and the other borrower. The borrower's initial or first period income is 10 and that of the lender is 40.

The first observation is that in the sharing system, contrary to the fixed interest case, plantings made by the lender and the borrower are not identical, which is in line with the analytical result of the previous section.This implies that the sharing case is not Pareto efficient (unless we assume a constant returns to scale technology): if the lender increases planting and the borrower decreases it, the total production and hence the total consumption of the farmer can be increased. However, the total planting (18.78) of the farmers (and also total consumption in two periods) is higher in the sharing system than that of the fixed interest system (16.66).

In order to check the Pareto optimality of the interest rate case we derive the Pareto efficiency conditions for the simple case of two farmers:

$$\text{Max } U_1(C_{11}) + b\, U_1(C_{21})$$

$$\text{s.t. } U_2(C_{21}) + b\, U_2(C_{22}) = U_2^{o}$$

C_{11} is the first period consumption of the first farmer where the first subscript refers to the time period and second to the farmer, [12] and

$$C_{12} = Y_1 + Y_2 - P_1 - P_2 - C_{11}$$

$$C_{22} = f(P_1) + f(P_2) - C_{21}$$

Y_1 and Y_2 are the initial endowments, P_1 and P_2 are the level of plantings, and $f(P_1)$ and $f(P_2)$ are outputs of the two farmers. The Langrangian function is:

$$\text{Max } Z = U_1(C_{11}) + b\, U_1(C_{21}) + g[(Y_1+Y_2-P_1-P_2-C_{11})$$

$$+ b\{f(P_1) + f(P_2) - C_{21}\} - U_2^{o}]$$

where g is the Lagrange multiplier. The first order conditions for the first farmer with respect to C_{11}, C_{21} and P_1 are

$$Z_{C11} = U_1'(C_{11}) - g\, U_2'(C_{12}) = 0 \qquad (4.1)$$

[12] Similarly we can define C_{21} and C_{22}.

TABLE 1(a)

$A = 8, d = 1.0, k = 0.5$

Sharing Case ($m = 0.4270$, $ra = 0.5530$)

	Y1	B	P	Y2*	P.B.**	C1	C2	C1+C2	U***
Bor	10	10.05	12.15	27.88	15.61	7.90	12.28	20.17	4.57
Len	40	10.05	(-)6.63	20.60	15.61	(-)23.32	36.21	59.52	6.73

TABLE 1(b)

Interest Case ($r = 0.3856$)

	Y1	B	P	Y2*	P.B.**	C1	C2	C1+C2	U***
Bor	10	7.50	8.333	23.09	10.39	9.17	12.70	21.868	4.75
Len	40	(-)7.50	8.333	23.09	(-)10.39	24.17	33.48	57.653	6.69

*Output.
**Pay Back (Principal and Interest or Profit).
***Utility.

$$Z_{C21} = b\,U_1'(C_{21}) - g\,b\,U_2\,(C_{22}) = 0 \tag{4.2}$$

$$Z_{P1} = -g\,[U_2'(C_{12}) + b\,U_2(C_{22})\,f'(P_1)] = 0 \tag{4.3}$$

Similarly, we can write the Lagrange function and the first order conditions for the second farmer:

$$\text{Max } L = U_2\,(C_{12}) + b\,U_2(C_{22}) + q[(Y_1+Y_2-P_1-P-C_{12})$$

$$+ b\{f(P_1)+f(P_2)-C_{22}\} - U_1^o]$$

$$L_{C12} = U_2'(C_{12}) - q\,U_1'(C_{11}) = 0 \tag{4.4}$$

$$L_{C12} = b\,U_2'\,(C_{22}) - q\,b\,U_1'(C_{21}) = 0 \tag{4.5}$$

$$L_{P2} = -q\,\{U_1'(C_{11}) + b\,U_1'(C_{21})\,f'(P_2)\} = 0 \tag{4.6}$$

where q is the Lagrange multiplier.

Equations (4.1) and (4.2) can be simplified to

$$\frac{U_1'(C_{11})}{U_1'(C_{21})} = \frac{U_2'(C_{12})}{U_2'(C_{22})} \tag{4.7}$$

On the other hand, equation (4.3) implies that

$$\frac{U_1'(C_{11})}{U_1'(C_{21})} = b\,f'(P_i) \tag{4.8}$$

Now we compare equations (4.7) and (4.8), the Pareto optimal conditions, with the two first order utility maximization conditions of the ith farmer in the interest case which are:

$$\frac{U'(C_{11})}{U'(C_{21})} = b\,f'(P_i) \tag{1.2a}$$

$$f'(P_i) = 1 + r \tag{1.2b}$$

It is clear that equations (1.2a) and (1.2b) satisfy both the Pareto optimal conditions. The same will be true for the other farmer.

We return to the discussion of numerical results. The second observation is that both the borrower and the lender in the fixed interest system plant up to the point where the marginal product is equal to 1+r. In the sharing case, however, though the lender plants up to the point where the marginal product is equal to 1+ra, the borrower goes beyond that level. For the borrower in the sharing case, at the optimal level of planting, the marginal product is less than both 1+ra and 1+r but greater than one. This can be summarized in Figure 2. The fact that the marginal product for the borrower in the sharing case is not equal to one is counter intutive. However, as investment/planting is higher in the sharing case, it establishes our contention that as the lenders share the residual income with the borrowers instead of getting a fixed percentage of the amount borrowed, investment would be generally higher in the sharing system.[13] To get further insights into this problem, we once again look at the maximization problem of the borrower:

$$\text{Max } U(Y_{1i} + B_i - P_i) + b\, U[f(P_i) - mB_i/P_i\{f(P_i) - P_i\} - B_i] \tag{4.9}$$

Suppose we fix mB_i/P_i (or the fraction of profit paid by the ith borrower to the lender) at m_i. The first order conditions for the borrower, with respect to P_i and B_i respectively, would then be

$$U'(C_{1i}) = bU'(C_{2i})[f'(P_i)-m_if'(P_i)+m_i] \tag{4.10}$$

$$U'(C_{1i}) = bU'(C_{2i})(1) \tag{4.11}$$

Dividing (4.8) by (4.9) and simplifying implies

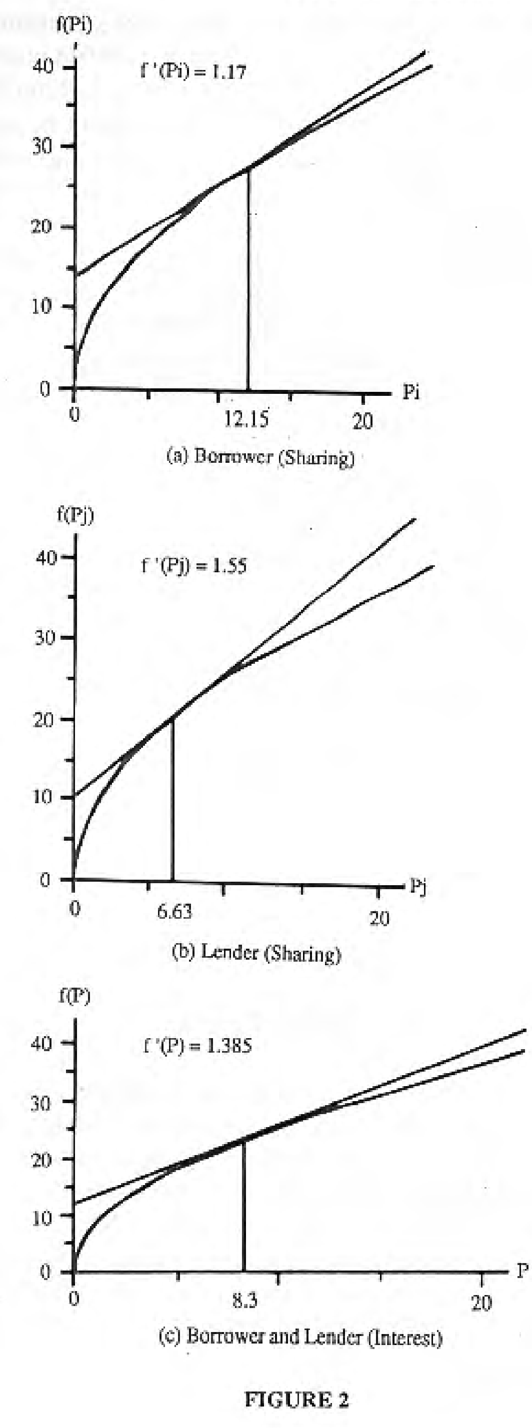
$$f'(P_i) - m_if'(P_i) + m_i = 1$$

$$\Rightarrow f'(P_i)(1-m_i) = (1-m_i)$$

$$\Rightarrow f'(P_i) = 1$$

If the share of profit paid by the borrower is independent of the amount of borrowing, then he carries out planting to the point where the marginal output is equal to one. As this is not true for our model, after a certain amount of planting is done, a further increase in planting increases the share of profit paid by the borrower relatively more than the increase in profit. In Figure 2 this critical level of planting is P_i*. Once the borrower reaches this level of planting, an agreement between the borrower and the lender to freeze the share of profit going to the lender, will induce the borrower to increase planting, making both of them better off. This result is very

[13] Haque and Mirakhor (1986a).



(a) Borrower (Sharing)

(b) Lender (Sharing)

(c) Borrower and Lender (Interest)

FIGURE 2

important as it can provide a guide line to the authorities responsible for determining the profit sharing ratios for the commercial banks and their customers. It also suggests that if, for different range of the amount of the loans, the profit sharing ratio is fixed, investment level will reach the point where marginal productivity is equal to one.

Another observation is that the rate of return (0.553) to the lender in the sharing system is higher than that of the fixed interest system (0.385). Finally, by looking at the total consumption of the farmer in the two periods, we observe that the lender is better off and the borrower is worse off in the sharing case compared to the fixed interest system. The reason is that in the sharing case the borrower does a higher level of planting, thus creating a higher demand for loans and a higher rate of return as well.

We now turn to investigate the general behaviour of different endogenous variables for different values of the parameters and the exogenous variables. We start with borrowing and planting. Figures 3 to 5 (Appendix) shows that borrowing is always higher in the sharing case for different values of b, k and A. This is due to the fact that the borrower in the sharing case does a higher amount of planting compared to both the lender in the sharing case and the borrower or the lender (as both do the same amount of planting) in the interest rate case [Figures 6 to 8, (Appendix)]. The least amount is always planted by the lender in the sharing case. As the borrower in the sharing case always plants a higher quantity, his consumption level for the first period is always the least [Figures 9 to 11, (Appendix)]. Similarly, as there is more lending in the sharing case, the lender also consumes less in the first period in the sharing case, compared to his counterpart in the interest case. In the second period, the borrower in the sharing case again consumes the least as the rate of return is too high for him [Figures 12 to 14, (Appendix)]. However, things are reversed for lenders in the two cases: the lender in the sharing case always consumes more in the second period. Total output produced (by both the borrower and the lender) is always higher in the sharing economy [Figures 15 to 17 (Appendix)]. Finally, we observe that the utility level of the borrower is always less than that of his counterpart in the interest rate system [Figures 18 to 20 (Appendix)]. The case is the reverse for lenders in the two systems. No qualitative change is observed when we decrease the difference in the initial incomes of the two farmers as shown in the Table.

V. Conclusion

We have developed a simple model to highlight some basic characteristics of Musharka finance. If all individuals are allowed to consume and produce without any element of uncertainty, and a decreasing returns to scale technology prevails, an economy under Musharka finance is inefficient (or Pareto inoptimal). However, the total investment (planting) is always higher in the sharing (Musharka) case, Also, borrowers in the sharing case are worse off compared to their counterparts in the interest case. This is because in the sharing case condition for optimality is such that demand for loanable funds is higher. In the present financial scenario this result implies, in general, a higher return for common depositors of commercial banks at the expense of producers/industrialists who are the ultimate users of these deposits.

In the interest rate case both the lender and the borrower plant (invest) up to the point where marginal productivity is equal to 1+r. In case of Musharka, where total planting is always higher, the lender plants up to the point where marginal productivity, is equal to $1+r_e$ (r_e is the effective rate of return to the lender and is always higher than the corresponding rate of interest in the interest rate system). The borrower on the other hand, does the highest amount of planting, though he never reaches the point where the marginal productivity is equal to one. However, if the sharing ratio is fixed (independent of the amount borrowed which is not generally the case), the borrower plants up to the point where marginal productivity is equal to one. This result is very interesting as it provides an important guide to the authorities responsible for fixing and announcing the profit and loss sharing ratio for financial intermediaries and the producers. As it is inappropriate to have one fixed sharing rate of profit for all sizes of loans, in line with our results and the discussion in the last section, we propose to have different fixed rates for different ranges of loans. The level of investment then, for one particular range, will be independent of the sharing rate. This implies that investment will be carried up to the point where marginal productivity is equal to one i.e., all projects will be undertaken which can at least cover the cost. This cost will, however, not include the fixed cost of borrowing or interest implying that investment will be higher under Musharka finance.

Applied Economics Research Centre
University of Karachi, and
Johns Hopkins University, USA

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APPENDIX

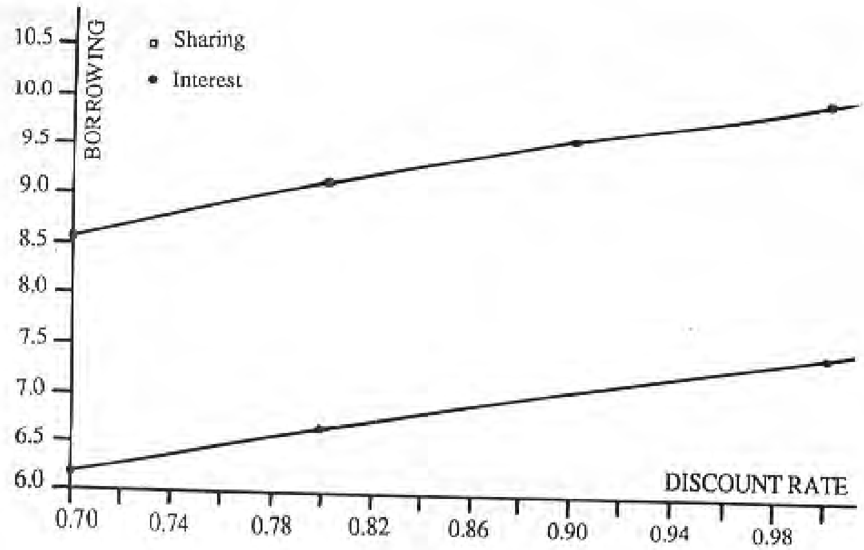

FIGURE 3

Borrowing with different Discount Rate

A = 8, k = 0.5

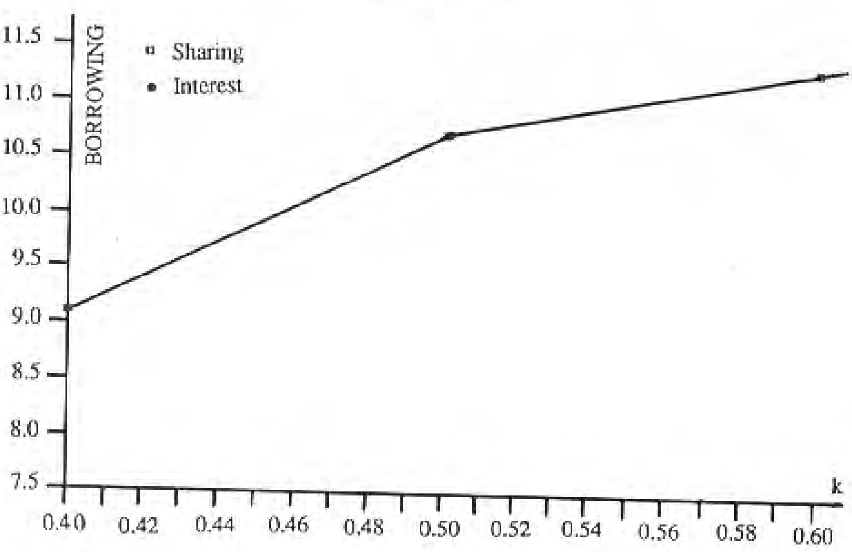

FIGURE 4

Borrowing with different Values of k

d = 1, A = 10

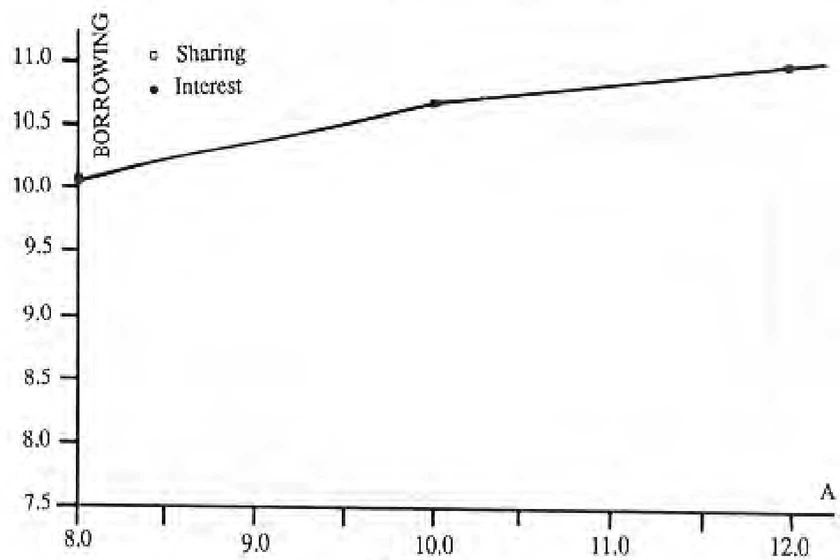

FIGURE 5
Borrowing with different Values of A
d = 1, k = 0.5

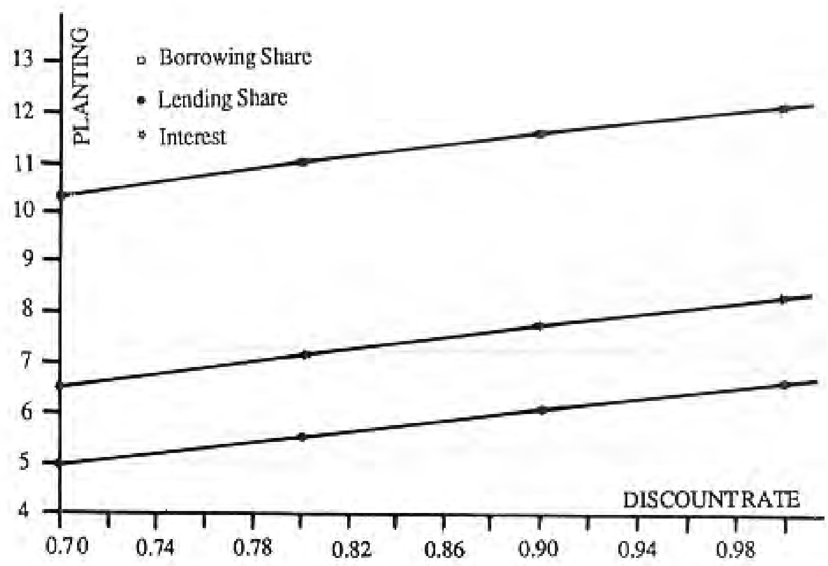

FIGURE 6
Planting with different Discount Rate
A = 8, k = 0.5

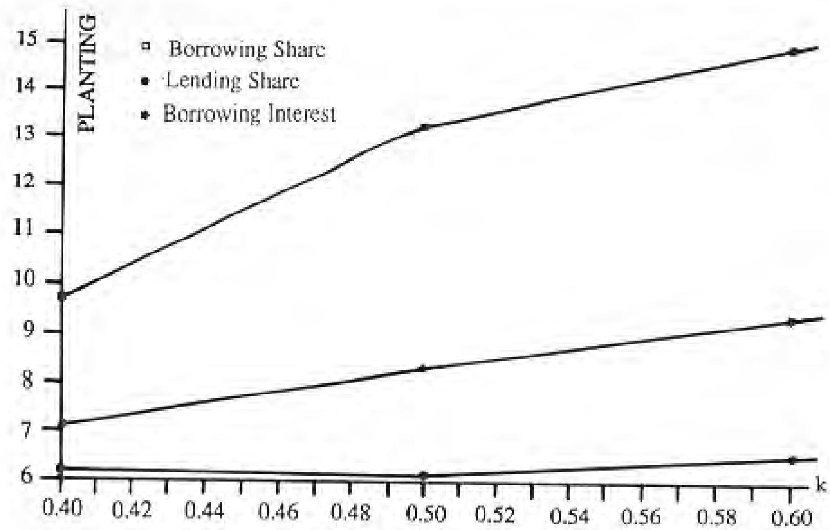

FIGURE 7

Planting with different Values of k

d = 1, A=10

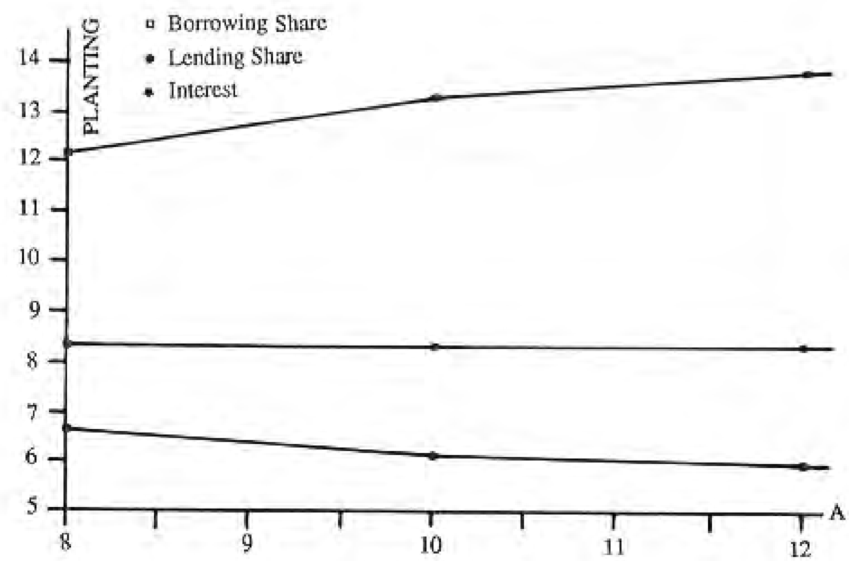

FIGURE 8

Planting with different Values of A

d = 1, k = 0.5

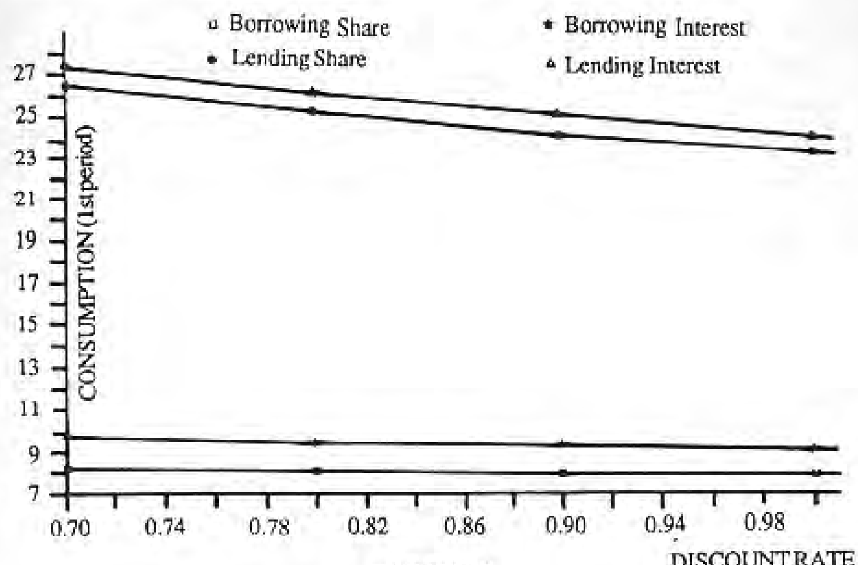

FIGURE 9

Consumption (1st period) with different discount rate

A = 8, k = 0.5

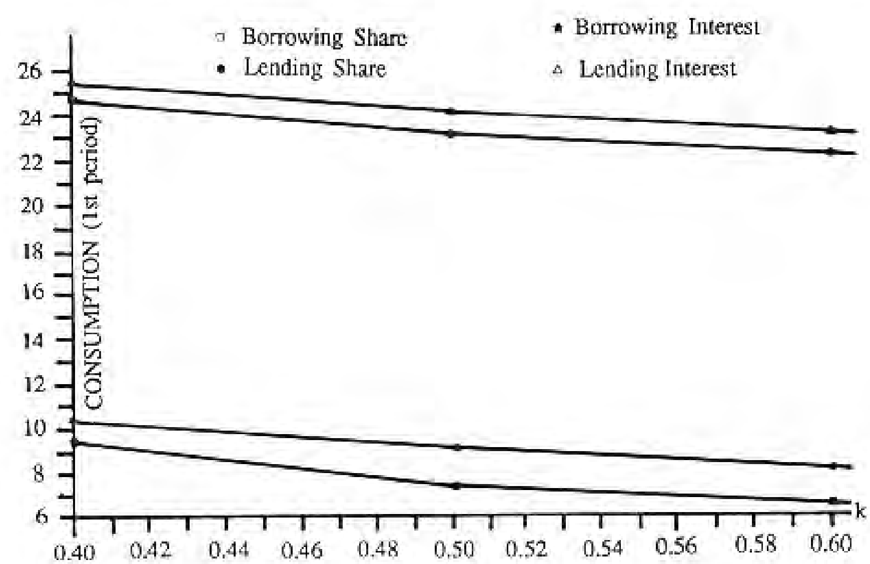

FIGURE 10

Consumption (1st period) with different Values of k

d = 1, A = 10

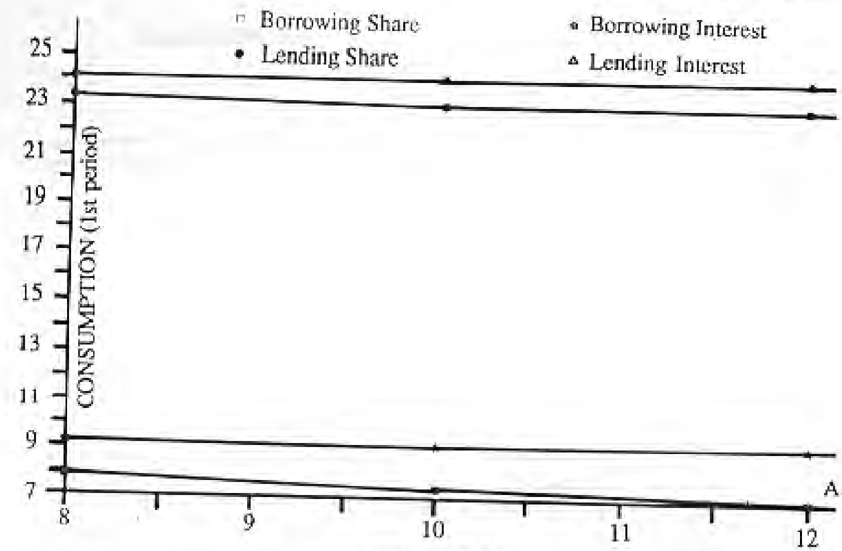

FIGURE 11

Consumption (1st period) with different Values of A

d = 1, k = 0.5

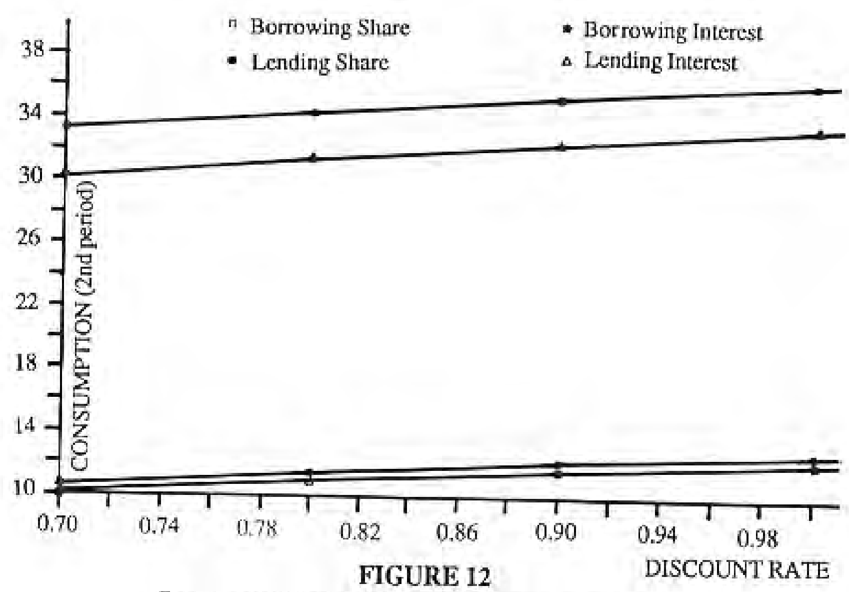

FIGURE 12

Consumption (2nd period) with different discount Rate

A = 8, k = 0.5

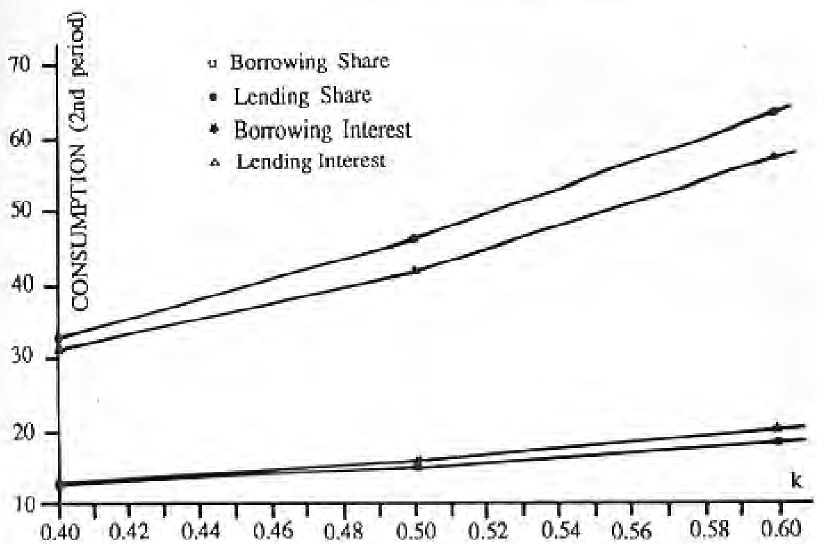

FIGURE 13

Consumption (2nd period) with different Values of k

d = 1, A = 10

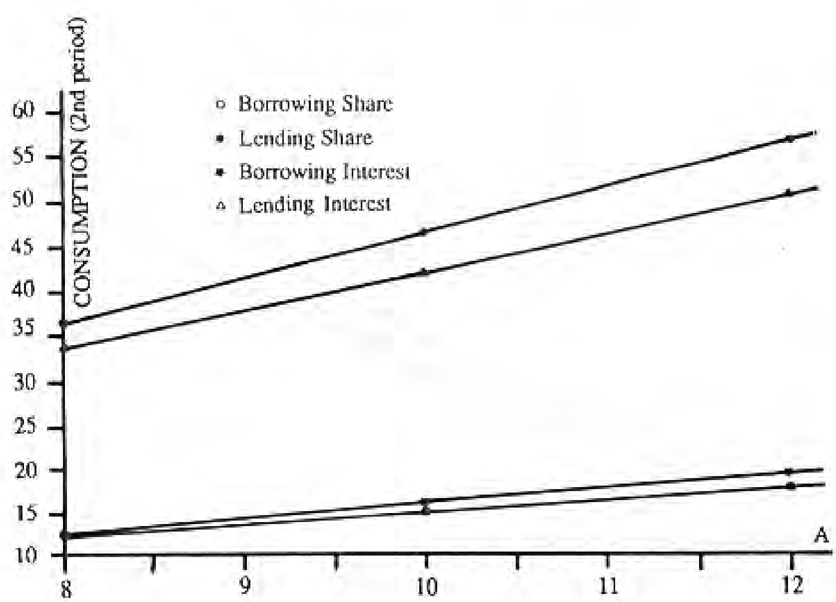

FIGURE 14

Consumption (2nd period with different Values of A

d = 1, k = 0.5

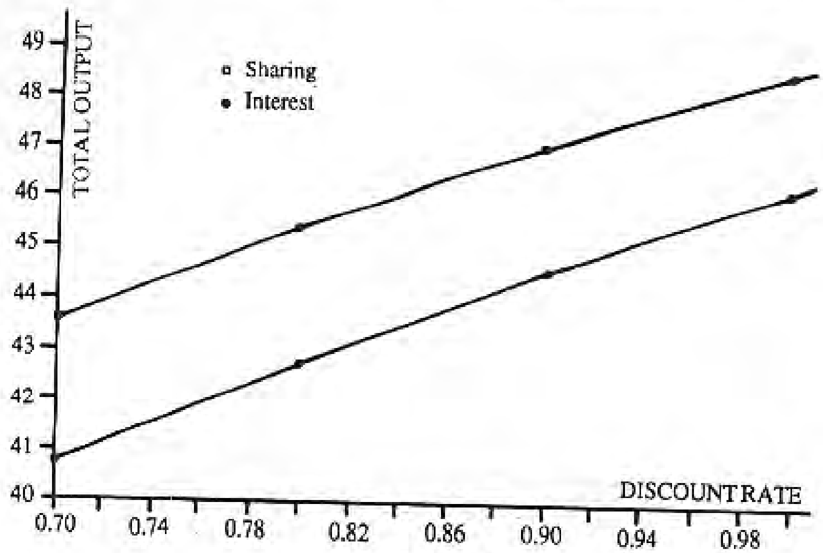

FIGURE 15

Total Output with different Discount Rate

A = 8, k = 0.5

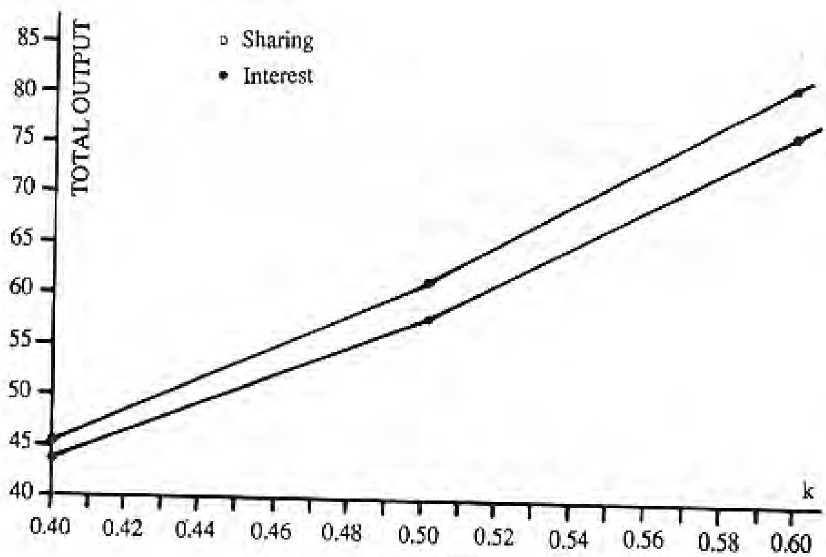

FIGURE 16

Total Output with different Values of k

d = 1, A = 10

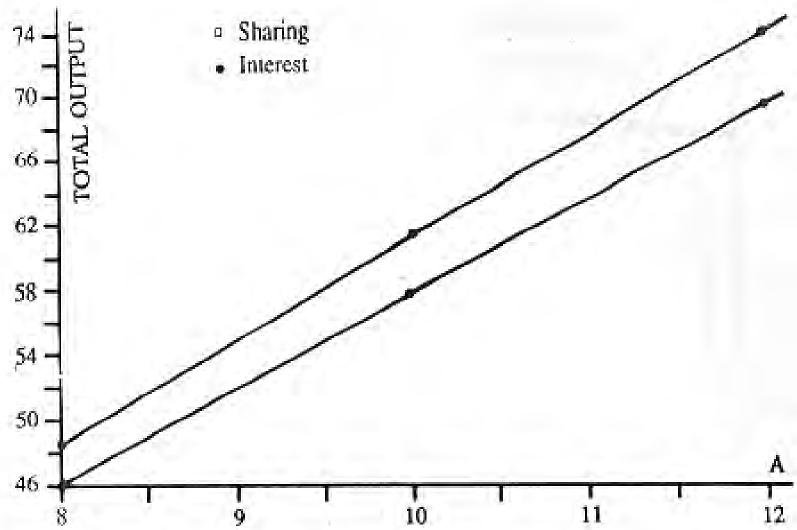

FIGURE 17

Total Output with different Values of A

d = 1, k = 0.5

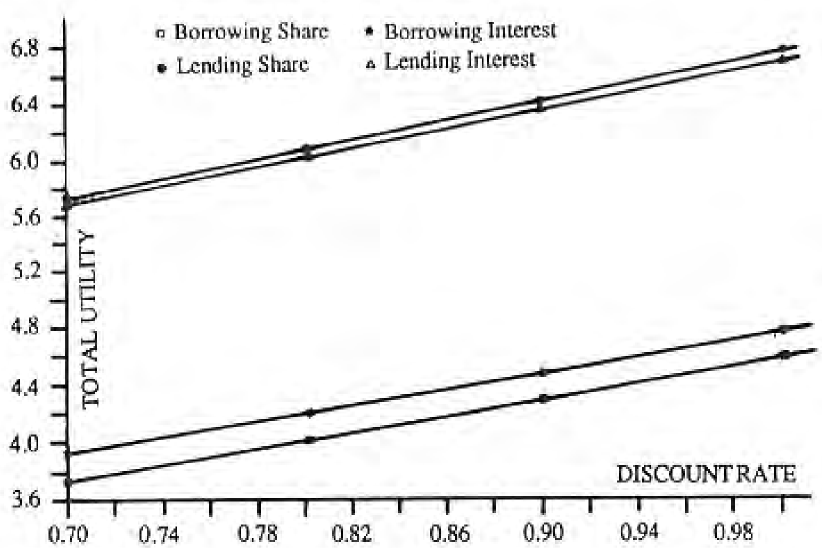

FIGURE 18

Total Utility with different Discount Rate

A = 8, k = 0.5

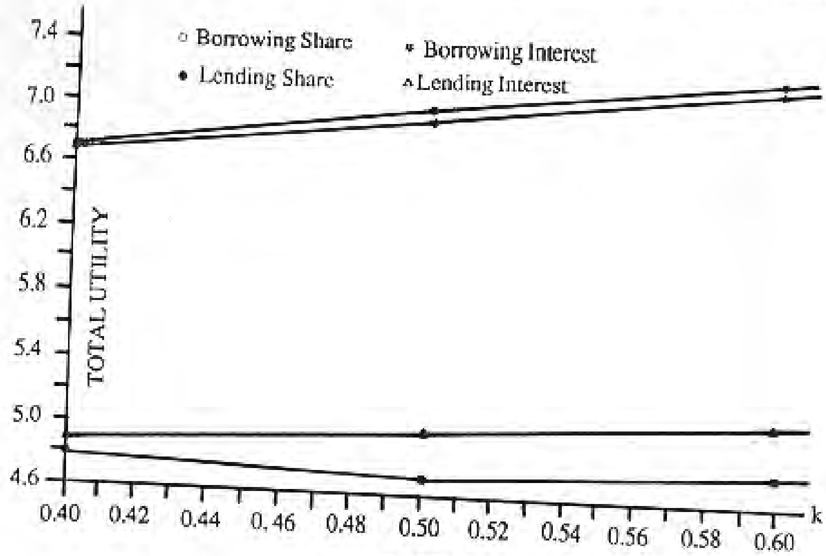

FIGURE 19

Total Utility with different Values of k

d = 1, A = 10

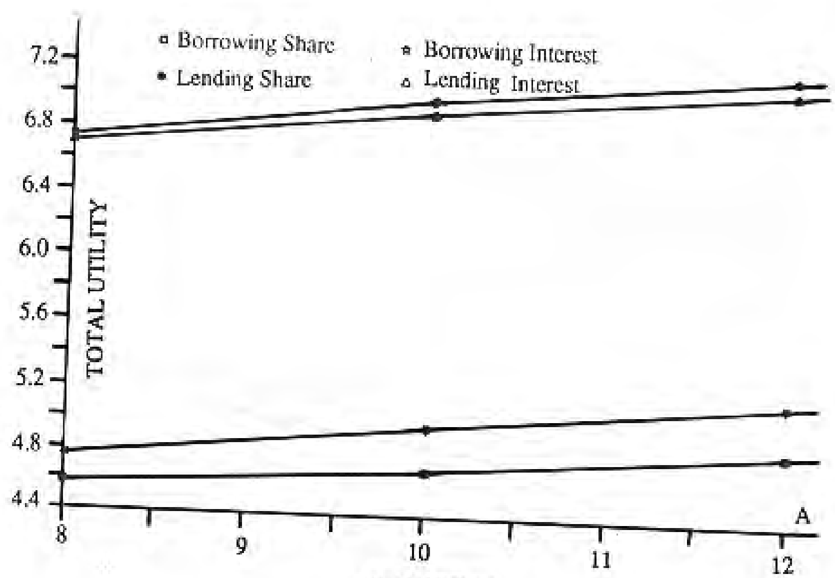

FIGURE 20

Total Utility with different Values of A

d = 1, k = 0.5