MODELLING GOVERNMENT SPENDING IN EXTERNAL FINANCING AND RESERVES WITH ESTIMATION FOR PAKISTAN

Mirza Faizan AHMED*, Shabbir AHMED* and Talha Ahmed SIDDIQUI**

Abstract

This paper augments a model developed for the government spending for a closed economy by introducing foreign loans, foreign reserves and exchange rates, which are of vital importance for budgeting in an open economy. It suggests that government expenditures are a function of taxes, domestic debt, foreign debt in domestic currency, foreign reserve in domestic currency, high-powered money, and real interest paid on domestic and foreign debt. All factors are found to have a positive impact on government expenditures in mathematical modelling, except for foreign reserves interest payments on domestic and foreign debt, which have a negative impact on the expenditures. The model is estimated by employing yearly data from 1976 to 2020 for Pakistan's economy and incorporating the impact of change in the exchange rate regime on government expenditures. The model, estimated with the Auto Regressive Distributed Lag (ARDL) model, suggests that current government expenditures depend on the current and previous period taxes, current domestic loans, current and earlier periods of foreign debt, current high-powered money, and foreign reserves. In addition, the empirical findings indicate that around 55 per cent of government expenditures have been financed through tax revenues, 21 per cent through high-powered money and 17 per cent through government bonds. The paper applies a dummy variable to measure the impact of the change in the exchange rate regime in 1999; it suggests that the change has positively impacted government expenditures. This paper concludes that the government has been preferring printing money and foreign loans, after taxes, for financing expenditures in Pakistan. At the policy level, the paper recommends that the government needs to rationalise the financing of its expenditures by focusing more on taxes. Further, it needs to avoid tapping foreign reserves held by the private sector.

Keywords: Government Expenditures, Foreign Reserve, Domestic Debt, and Exchange Rate.

JEL Classification: L16, F63, J24, F36, C31.

I. Introduction

Government expenditures contribute to economic output. As Keynesian economics theorises, increasing government expenditures can lead to economic growth, and its intervention can stabilise the economy. Such as Devarajan, et al., (1996) estimated that

^{*} Assistant Professor, **Lecturer, Department of Economics and Management Sciences, NED University of Engineering and Technology, Karachi, Pakistan.

an increase in government expenditures has a positive and significant impact on economic growth in 43 developing economies. However, any such intervention needs to be backed by revenues or financing. Typically, government revenues are derived from its tax base. However, in low-income countries, income taxation-seigniorage is a comparatively less distortionary way of financing public spending [Bose, et al., (2007)].

In addition, government expenditures can also be financed through borrowings. Interestingly, a part of financing in terms of loans is an expense, such as interest or cost of financing. The interest expense makes no addition to the tangible government expenditures; instead, it makes a way out towards its lender.

There is also a role of high-powered money issued by the central bank against its government bond holdings. They buy bonds and issue currency against them. These are a few sources to accumulate funds for making expenditures by the government. These sources, also called determinants of government expenditures, create its size. These determinants include both fiscal and monetary variables. Christ (1979) highlights that fiscal variables and open market operations strongly influence the effect of a given change in the high-powered money stock and public spending and revenues.

However, Walsh (2009) developed a model for government expenditures by considering these factors. It compares two budget constraints: one is the government, and the other one is the central bank. The determinants of government expenditures include taxes, interest on debt, additions—in debt, high-powered money, inflation, and gross domestic output. The model best suits a closed economy with a major share of debt comprised of internal sources of the economy. However, if we take foreign debt, such as open economies, additional foreign reserves and exchange rate determinants for government expenditures would be added.

The central bank issues high-powered money against the foreign reserves they get against foreign loans. But that is not the only source of reserves rather, the reserves are also received from the private sector, which are also converted into high-powered money. Therefore, increasing private-sector reserves limits government expenditures' capacity by seigniorage. In addition, these reserves have to be transformed by the exchange rate, which brings another determinant for government expenditures.

Hence, this paper augments the model Walsh (2009) developed by introducing foreign loans, foreign reserves and exchange rates. It also uses the case of Pakistan for estimating the model. The determinants in the augmented model for government expenditures are a function of government revenues or taxes, interest payments on bonds held by the private sector, interest on government foreign debt, change in private and foreign bond holdings, high-powered money, foreign reserves held by the central bank, and exchange rate. The variables are taken as a percentage of real GDP.

This paper follows Section II, which highlights the previous literature. Section III focuses on the derivation of empirical relation; while Section IV highlights the methodology and the estimation techniques that are used in the paper. Section V discusses the results of the model and the final Section VI draws conclusions and policy implications.

II. Literature Review

Tax revenues generally cover government expenditures. If the revenues do not cover spending, the government takes loans from other sources or prints the new currency. A government budget constraint occurs when it faces challenges raising sufficient funds to cover its increasing expenditure through taxes. The government budget constraint is an accounting identity that connects the fiscal authority's choices of expenditure, taxes, and borrowing time with the monetary authority's choices of money growth or nominal interest rate. There are two strands of literature on government budget constraints: one that focuses on the composition and problems associated with it and the other that discusses various factors that impact government expenditure and revenue.

Sargent and Wallace (1981) mark the beginning of the contemporary treatment of government budget constraints. Their study showed a higher money supply could be an important initiative to meet fiscal imbalances. Similarly, Leeper (2005), Sims (1994), and Cochrane (2001) discussed the concept of the fiscal theory of the price level (FTPL), which indicates that the government's intertemporal budget equation is an equilibrium condition, not a constraint. The literature further claimed that the equilibrium price level is crucial for maintaining a balanced government's intertemporal budget, given the nominal debt stock.

Bohn (1991) calculated the difference between government-owned assets and government liabilities at market prices and estimated government accounting practices related to budget deficits. The author suggested that the accrual basis of accounting may be used for national income accounting. In addition, the study also demonstrates that government debt is economically relevant for the intergeneration redistribution of resources.

Ferretti (2000) analysed how fiscal regulations affect budgetary choices when a government has room for creative accounting. It draws attention to the trade-offs between deficit bias and cyclical stabilisation. Similarly, there is also a trade-off between window dressing and real fiscal adjustment. The author indicates that a strict rule encourages creative accounting and less fiscal adjustment, although the budget still has some cyclical shock response capacity. The model has plausible implications for regulations that are more stringent on nations with a history of fiscal irresponsibility and less transparent budgets. Its drawbacks include the inability to endogenise transparency and examine incentives for inventive accounting across extended periods.

Hall and Sargent (2011) showed the measurement challenges associated with budget constraints. They showed that these challenges result from the limitations on how governments and central banks may account for government debt. Official debts priced to market are the focus of the macroeconomic constraints on budgets. Yet, official accounts often report par values are inadequate at accounting for coupon payments, let alone default premia. Even the strongest government accounting systems, like those of the United States and the United Kingdom, can reflect government debt measurements that significantly differ from the targets of a macroeconomist's government budget constraint.

Recent history indicates that large shocks can cause fiscal rules to become pro-cyclical, potentially offsetting monetary policy. When policy of managing interest rate loses effectiveness, fiscal policy is more effective in overcoming budgetary imbalance. This has led to policymakers advocating for monetary-fiscal policy coordination. Similarly, the government budget constraint links monetary and fiscal decisions to future policy variables, creating a complex intertemporal dimension that affects macro policy actions, as Leeper and Nason (2005) discussed in the literature.

A recent study by Reichlin, et al., (2023) proposed an empirical framework for studying adjustment dynamics through primary deficits, yields, returns, debt market value, and inflation in the context of the European monetary Union. The framework uses the general government budget constraint as an identity and a Structural Vector Autoregressive (SVAR) model to obtain dynamics. The long-term budget constraint can be understood in a monetary union with different fiscal authorities and a single monetary authority. The nature of the fiscal-monetary interaction depends on whether unexpected monetary policy affects the short-term interest rate (conventional) or the long end of the yield curve (unconventional). The study suggests that the European monetary union may be considered for the coordination of fiscal and monetary policy to ensure a coherent monetary and fiscal stance at the aggregate level.

The empirical literature has also discussed external debt as a major factor that impacts government spending, hence impacting government budget constraints. Mahdavi (2004) investigated the effects of external public debt burden on public spending in 47 developing countries over the period 1972 to 2000. Their study found a negative relationship between external debt and the government's expenditure. In comparison, Friedman (1978) showed that the change in government revenue has a major impact on government expenditure. The two have a one-way causality, running from government revenue to government expenditure.

Dornbusch (1984) highlighted the impact of external borrowings on government budget constraints. He showed that developing nations have a difficult time controlling their fiscal deficits when they are financed by loans from abroad that need to be paid back. Countries face various problems due to this debt crisis, and they have to make a choice between politically challenging measures like raising taxes, cutting subsidies, shrinking the public administration and accruing wage arrears, or printing money and issuing domestic debt. Mao, et al., (2020) examined the impact of policy regime changes on government spending. The study showed that government expenditure multipliers can be lower under passive monetary policy due to high debt and high tax rates.

This literature highlights some empirical studies carried out to identify sources and their impact on government expenditures. Walsh (2009) has also developed a model in this regard by using the government and central banks' budgets for a closed economy. However, there is a gap in determining the financing sources for government expenditures theoretically and mathematically under the modelling framework of Walsh (2009) for

an open economy. The novelty of this paper is to develop the model by incorporating foreign reserves, foreign loans, and exchange rates, which are significant for an open economy, and estimate the model for public spending in Pakistan.

III. The Model

Walsh (2009) develops a model for the determinants of government expenditures, which is extended in this paper by incorporating foreign loans, foreign reserves and exchange rates. Let's assume the government's budget constraint consists of two major factors: Taxes and total borrowings BtT. Interest on borrowings reduces the expenditures in Equation (1).

$$G_{t} + i_{t,t} B_{t,t}^{T} = T_{t} + (B_{t}^{T} - B_{t,t}^{T})$$
 (1)

Where superscript 'T' shows 'total' and subscript 't' represents time, B reflects the government Debt (Bonds issued by the government and foreign debt), and T indicates the total taxes that the government collects. While i represent the interest amount of government debt.

Total government debt B^T comprises government bonds held by the central bank, B^C , and government bonds held by the public, B^P , and government foreign debt, which is converted into local currency by multiplying the exchange rate in Equation (2), i.e. eB^F

$$B_t^T = B_t^P + B_t^{CB} + e_t B_t^T \tag{2}$$

This implies that in Equation (3)

$$B_{t-1}^T = B_{t-1}^P + B_{t-1}^C + e_{t-1}B_{t-1}^F$$
 (3)

Putting the values of B_t^T and B_{t-1}^T from Equations (2) and (3) in Equation (1), we get Equation (4)

$$G_{t} + i_{t-1}(B_{t-1}^{P} + B_{t-1}^{C} + e_{t-1}B_{t-1}^{F}) = T_{t} + (B_{t}^{P} + B_{t}^{C} + e_{t}B_{t}^{F} - B_{t-1}^{P} - B_{t-1}^{C} - e_{t-1}B_{t-1}^{F})$$
(4)

Transforming the Equation (4) we get Equation (5)

$$G_{t} + i_{t-1}(B_{t-1}^{P} + B_{t-1}^{C} + e_{t-1}B_{t-1}^{F}) = T_{t} + (B_{t}^{P} - B_{t-1}^{P}) + (B_{t}^{C} - B_{t-1}^{C}) + (e_{t}B_{t}^{F} - e_{t-1}B_{t-1}^{F})$$
 (5)

On the other side, the central bank's budget constraint can be written in terms of government bonds held by central bank, high-powered money issued against the bonds and interest received on the bonds are followed by in Equation (6).

$$B_{t}^{C} - B_{t,l}^{C} = i_{t,l} B_{t,l}^{C} + (H_{t} - H_{t,l})$$

$$(6)$$

This budget constraint, as used by Walsh (2009), is extended as high-powered money is not only issued against buying government bonds but also against the increase in foreign reserves by the central bank. Here, we represent foreign exchange reserves by 'S'. Reserves include government external or foreign debt ' B^F ' and reserves held by private sector 'X'. The exchange rate is shown by 'e'. Therefore, the augmented central bank budget constraint in domestic currency will be transformed from Equation (6) to Equation (7)

$$(e_t S_t - e_{t-1} S_{t-1}) + (B_t^C - B_{t-1}^C) = i_{t-1} B_{t-1}^C + (H_t - H_{t-1})$$
(7)

As the foreign reserves can be written as follows in Equation (8),

$$S_{\cdot} = X_{\cdot} + B_{\cdot}^{F} \tag{8}$$

This relationship with some mathematical transformation turns Equation (7) into Equation (9)

$$(B_t^C - B_{t-1}^C) = i_{t-1}B_{t-1}^C + (H_t - H_{t-1}) - (e_t X_t - e_{t-1} X_{t-1}) - (e_t B_t^F - e_{t-1} B_{t-1}^F)$$
(9)

Putting the value of $(B_t^C - B_{t-1}^C)$ from Equation (9) in Equation (5), we get the following Equation (10) with some mathematical operations,

$$G_{t} + i_{t-1}(B_{t-1}^{p} + e_{t-1}B_{t-1}^{F}) = T_{t} + (B_{t}^{p} - B_{t-1}^{p}) + (H_{t} - H_{t-1}) - (e_{t}X_{t} - e_{t-1}X_{t-1})$$
(10)

As these variables are in absolute values and can be converted into as percentage of GDP. Hence, the above Equation divided by the nominal GDP, $P_t Y_t$ where P_t as the overall price level and Y_t as real GDP, we get the following Equation (11),

$$g_{t}^{+}i_{t,l}\left[\frac{b_{t,l}^{p}}{(l+\pi_{t})(l+\lambda_{t})} + \frac{e_{t,l}b_{t,l}^{F}}{(l+\pi_{t})(l+\lambda_{t})}\right] = t_{t}^{+}\left[b_{t}^{p} - \frac{b_{t,l}^{p}}{(l+\pi_{t})(l+\lambda_{t})}\right] + \left[h_{t} - \frac{h_{t,l}}{(l+\pi_{t})(l+\lambda_{t})}\right] - \left[e_{t}x_{t} - \frac{e_{t,l}x_{t,l}}{(l+\pi_{t})(l+\lambda_{t})}\right]$$
(11)

Variables that are converted from capital letters to small letters show the variable as percentage of nominal GDP, π_t is the overall inflation rate and λ_t is the real GDP growth rate in the economy.

The conversion from an actual variable to a variable as a percentage of nominal GDP at time 't' and 't-1' are as follows in Equations (12) and (13):

 $^{^{1}}$ Reserves only include foreign currency, gold, other assets against which central bank issues high powered money.

$$\frac{B_t^P}{P_t Y_t} = b_t^P \text{ for time 't'}$$
 (12)

$$\frac{B_{t-l}^{P}}{P_{t}Y_{t}} = \frac{B_{t-l}^{P}}{P_{t-l}Y_{t-l}} \frac{P_{t-l}Y_{t-l}}{P_{t}Y_{t}} = \frac{b_{t-l}^{P}}{(1+\pi_{t})(1+\lambda_{t})}, \text{ for time 't-1'}$$
(13)

Real interest rate \overline{r}_{t-1} defined as the interest rate at 't-1' discounted by the nominal GDP at 't-1' comes as follows in Equation (14)

$$r_{t-1} = \frac{1 + i_{t-1}}{(1 + \pi)(1 + \lambda)} - 1 \tag{14}$$

Putting the value of real interest rate from Equation (14) in Equation (11) with a few mathematical operations, we get Equation (15)

$$g_{t}^{+}r_{t-l}b_{t-l}^{P}+i_{t-l}\frac{e_{t-l}b_{t-l}^{F}}{(1+\pi_{t})(1+\lambda_{t})}-\frac{e_{t-l}x_{t-l}}{(1+\pi_{t})(1+\lambda_{t})}=t_{t}^{+}[b_{t}^{P}-b_{t-l}^{P}]+\left[h_{t}-\frac{h_{t-l}}{(1+\pi_{t})(1+\lambda_{t})}\right]-e_{t}x_{t}$$
(15)

Using Equation (8) we can conclude that the private foreign reserves holdings as a percentage of nominal GDP can be written as in Equations (16) and (17)

$$x_t = s_t - b_t^F \tag{16}$$

$$x_{t-1} = s_{t-1} - b_{t-1}^F (17)$$

Putting the values of x_t and x_{t-1} from Equations (16) and (17) and using the formula of real interest rate as shown in Equation (14) in Equation (15), we get Equation (18)

$$g_{t}^{+}r_{t-l}b_{t-l}^{P} + r_{t-l}e_{t-l}b_{t-l}^{F} = t_{t}^{+} + [b_{t}^{P} - b_{t-l}^{P}] + [e_{t}b_{t}^{F} - e_{t-l}b_{t-l}^{F}] + \left[h_{t}^{-}\frac{h_{t-l}}{(l+\pi_{t})(l+\lambda_{t})}\right] - \left[e_{t}s_{t}^{-} - \frac{e_{t-l}f_{t-l}}{(l+\pi_{t})(l+\lambda_{t})}\right]$$
(18)

To make all the variables flow variables, we can transform Equation (18) as follows in Equation (19)

$$g_{t}^{-}=t_{t}^{+}[b_{t}^{p}-b_{t-l}^{p}]+[e_{t}b_{t}^{p}-e_{t-l}b_{t-l}^{p}]+[h_{t}-h_{t-l}]-[e_{t}s_{t}-e_{t-l}s_{t-l}]-r_{t-l}b_{t-l}^{p}-r_{t-l}e_{t-l}b_{t-l}^{p}+[h_{t-l}-e_{t-l}s_{t-l}]*\left[l-\frac{1}{(l+\pi_{t})(l+\lambda_{t})}\right]$$
(19)

Let $[h_{t-l}-e_{t-l}s_{t-l}]^{\frac{1}{2}} = \frac{1}{(1+\pi_{l})(1+1)}$ is equal to an error term in Equation (19), it gives Equation (20)

$$g_{t} = t_{t} + (b_{t}^{P} - b_{t-l}^{P}) + (e_{t}b_{t}^{F} - e_{t-l}b_{t-l}^{F}) + (h_{t} - h_{t-l}) - (e_{t}s_{t} - e_{t-l}s_{t-l}) - r_{t-l}b_{t-l}^{F} - r_{t-l}e_{t-l}b_{t-l}^{F} + \mu t$$
(20)

All the variables will be as a percentage of nominal GDP unless specified. The above Equation is the final Equation of the model. It explains that government expenditures are a function of government revenues or taxes, the net change in private bond holdings, foreign debt, high-powered money, foreign reserves held by the central bank, and the exchange rate. All the variables hypothesis to have a positive effect on government expenditures except for the changes in foreign reserves, which reduces government expenditures. In addition, government expenditures are reduced due to interest payments on bonds held by the private sector and interest on government foreign debt.

The model, as concluded in Equation (20), can be written as follows to in terms of an econometric model in Equation (21):

$$g_{t} = \alpha_{0} + \alpha_{1}t_{t} + \alpha_{2}(b_{t}^{P} - b_{t-1}^{P}) + \alpha_{3}(e_{t}b_{t}^{F} - e_{t-1}b_{t-1}^{F}) + \alpha_{4}(h_{t} - h_{t-1}) - \alpha_{5}(e_{t}s_{t} - e_{t-1}s_{t-1}) - \alpha_{6}r_{t-1}b_{t-1}^{F} - \alpha_{7}r_{t-1}e_{t-1}b_{t-1}^{F} + \mu_{t}$$
(21)

Where value of eight parameters from α_0 to α_7 determines the average share of each factor on the government expenditures. The methodology used to estimate the model for a case of Pakistan's economy as a developing country and an open economy is discussed in the upcoming section.

IV. Estimation Methodology

The model developed in this paper is estimated for Pakistan's economy as a case from open economies. The model has time series variables for which we have developed standard methodologies. The annual time series data from 1976 to 2020 for Pakistan is collected from the handbook of statistics published by the State Bank of Pakistan and the Economic Survey of Pakistan.

All the data are converted from actual variables to variables as a percentage of nominal GDP using the formula given in Equation (12). Apart from these variables, the real interest rate is computed by using Equation (14). The estimation procedure begins with a unit root test using the Augmented Dickey-Fuller (ADF) test. Based on the unit-root test results, a suitable estimation model is proposed: Autoregressive Distributed Lag Model (ARDL).

The ARDL model has multiple advantages compared to other time series models [Pesaran, et al., (2001) and Pesaran, et al., (1999)]. The classical ARDL approach can be used for data of short time horizons [Haug (2002)]. ARDL model can be utilised when the variables are stationary and integrated at most of order one, i.e. I(0) or I(1).

1. Autoregressive Distributed Lag Bounds Test

The ARDL bound test co-integration is applied to measure the impact of the longrun relationship between government expenditures and its determinants as derived in the model above. The general ARDL bound test co-integration model is as follows:

$$\begin{split} \Delta g_{t} &= \alpha_{0} + \alpha_{1} g_{-l} + \alpha_{2} t_{t-l} + \alpha_{3} b_{t-l}^{p} + \alpha_{4} e_{t-l} b_{t-l}^{f} + \alpha_{5} h_{t-l} + \alpha_{6} e_{t-l} s_{t-l} + \alpha_{7} r_{t-l} b_{t-l}^{p} \\ &+ \alpha_{8} r_{t-l} e_{t-l} b_{t-l}^{f} + \sum \beta_{l} g_{t-l} + \sum \beta_{2} \Delta t_{t-l} + \sum \beta_{3} \Delta b_{t-l}^{p} + \sum \beta_{4} \Delta e_{t-l} b_{t-l}^{f} \\ &+ \sum \beta_{5} \Delta h_{t-l} + \sum \beta_{6} \Delta e_{t-l} s_{t-l} + \sum \beta_{7} \Delta r_{t-l} b_{t-l}^{p} + \sum \beta_{8} \Delta r_{t-l} e_{t-l} b_{t-l}^{f} + \varepsilon_{t} \end{split}$$

Where, g represents the government expenditures, t indicates tax revenues, b^p reflects government bond holdings by the public, eb^f shows foreign bond holdings in terms of domestic currency, h reflects high powered money, es shows foreign reserves in terms of domestic currency and rb^p reflects the real interest paid on government bonds to the public and reb^f shows real interest paid on foreign bonds.

The lag selection is based on the Akaike Information Criterion (AIC). For the stability test, post estimation test of CUSUM and CUSUMSQ are applied as suggested by Brown et al., (1975).

V. Empirical Results

Government expenditures (g_t) in any given year as modelled above are derived to be a function of tax revenues (t_t) , change in government bond holdings by the public $[\Delta b_t^P = (b_t^P - b_{t-1}^P)]$, change in foreign bond holdings in terms of domestic currency $[\Delta e_t b_t^F = (e_t b_t^F - e_{t-1} b_{t-1}^F)]$, change in high-powered money $[\Delta h_t = (h_t - h_{t-1})]$, change in foreign reserves in terms of domestic currency $\Delta e_t s_t = (e_t s_t - e_{t-1} s_{t-1})$, and real interest paid on government bonds to the public $(r_{t-1} b_{t-1}^P)$ and real interest paid on foreign bonds $(r_{t-1} e_{t-1} b_{t-1}^F)$ in the preceding year.

TABLE 1

Descriptive analysis of government expenditures and its determinants

Statistics	g_{t}	$t_{_t}$	$\Delta b_{_t}^{_P}$	$\Delta e_t b_t^P$	$\Delta h_{_t}$	$\Delta e_t s_t$	$r_{t-1}b_{t-1}^P$	$r_{t-l}e_{t-l}b_{t-l}^F$
Mean	0.165	0.090	0.008	0.002	0.005	0.001	-0.009	-0.011
Median	0.164	0.088	0.007	0.001	0.007	-0.002	-0.009	-0.011
Max	0.203	0.114	0.052	0.081	0.037	0.033	0.009	0.008
Min	0.137	0.077	-0.030	-0.052	-0.032	-0.035	-0.027	-0.032
Std. Dev.	0.016	0.008	0.019	0.029	0.017	0.015	0.010	0.011
Skewness	0.263	1.360	0.171	0.217	-0.305	0.218	-0.085	0.011
Kurtosis	2.315	5.140	2.677	2.795	2.604	2.656	2.156	2.088

The above descriptive analysis shows that the average government expenditures have been 16.5 per cent of total nominal GDP, ranging from 13.7 per cent to 20.3 per cent. Similarly, the tax to nominal GDP has been 9 per cent, and changes in public bond holding and foreign bonds have been less than 0.8 per cent and 0.2 per cent, respectively. Changes in high-powered money and foreign reserves in terms of domestic currency have been 0.5 per cent and 0.1 per cent, respectively. The average real interest rate paid on government bonds held by the public and the real interest paid on foreign debt have been -0.9 per cent and -1.1 per cent, respectively. The reason for negative real interest is that since 1976, there have been only ten years in which the real interest rate (nominal interest rate adjusted by growth in nominal GDP) has been positive. In other words, the nominal GDP has been growing more rapidly than the nominal interest rate. The changes in foreign reserves and foreign bonds in domestic currency have been highly volatile among all the variables. On the contrary, the tax revenues and government expenditures are the least volatile. Here, we can deduce that fiscal effort to finance government expenditures has been sparse, and the fiscal deficit is covered by one or the other remaining determinants of government expenditures.

Table 2 highlights the unit root test for checking the stationary of the dependent variable and regressors. Only one regressor and the dependent variable are found to be integrated into order one. All the variables are tested using Augmented Dicky Fuller (ADF) test. However, the stationary results are consistent at 'intercept' and 'trend and intercept'. Further, the Dicky Fuller (DF) and Phillips Perron (PP) tests of stationary also give consistent results.

TABLE 2
Results of Unit-Root Test

Variable -	At	level	At 1st d	Decision		
variable –	t-values	Probability	t-values	Probability	Decision	
Government Expenditures	0.469	0.812	-7.688	0	I(1)	
Tax Revenues	0.445	0.806	-7.570	0	I(1)	
Government Bond Holding by Public	-3.982	0	-	-	I(0)	
Foreign Bond Holding	-5.108	0	-	-	I(0)	
High Power Money	-5.496	0	-	-	I(0)	
Chang in Foreign Reserve	-6.684	0	-	-	I(0)	
Real Interest Paid Government Bond to Public	-2.907	0.005	-	-	I(0)	
Real Interest Payment on foreign Bond on preceding year	-2.954	0.004	-	-	I(0)	

With these stationary results, we can select the ARDL model for estimation. As we have introduced foreign loans, foreign reserves, and exchange rates in the model, another consideration we need to cater to is the change in exchange rate regime by the State Bank of Pakistan in 1999 from a fixed to a market-determined exchange rate. This is likely to change the impact of the exchange rate in the model. Therefore, in the estimation, we added a dummy variable keeping value '0' from 1976 to 1999 and value '1' from 2000 onwards to capture the possible impact of change in the exchange rate regime.

In the estimations, it is found that the majority of variables become insignificant and did not produce meaningful results after multiple attempts by changing the dynamic and fixed variables in the estimation of the ARDL model and applying the dummy variable with foreign loan, foreign reserves, and real interest on foreign loans. Two of the more prominent insignificant variables are real interest paid on government bonds to the public and real interest paid on foreign bonds in the preceding year due to a problem of multicollinearity as they have a 95 per cent correlation between them, Table 3. If we drop any of the two variables, the results still seem misleading or meaningless. Therefore, the estimation is carried out after variables fall. The final model estimated is as follows, with a maximum of three lags of regressors and dependent variables as follow in Equation (22).

$$g_{t} = \alpha_{0} + \alpha_{1} t_{t} + \alpha_{2} (b_{t}^{P} - b_{t-1}^{P}) + \alpha_{3} (e_{t} b_{t}^{F} - e_{t-1} b_{t-1}^{F}) + \alpha_{4} (h_{t} - h_{t-1}) - \alpha_{5} (e_{t} s_{t} - e_{t-1} s_{t-1}) + \mu_{t}$$
(22)

Using the Schwarz criterion for automatic lag selection, the following model comes up in Equation (23).

$$g_{t} = \alpha_{0} + \beta_{1} g_{t-1} + \alpha_{11} t_{t-1} + \alpha_{12} t_{t} + \alpha_{2} \Delta b_{t}^{P} + \alpha_{3} \Delta e_{t} b_{t}^{F} + \alpha_{4} \Delta h_{t} - \alpha_{5} \Delta e_{t} s_{t} + \alpha_{61} \Delta e_{t-2} b_{t-2}^{F} * d_{t-2} + \alpha_{62} \Delta e_{t-1} b_{t-1}^{F} * d_{t-1} + \alpha_{63} \Delta e_{t} b_{t}^{F} * d_{t} + \mu_{t}$$

$$(23)$$

TABLE 3

Correlation Matrix of Dependent Variables and Regressors (in Percentage)

	g_{t}	$t_{_t}$	$\Delta b_{_t}^{_P}$	$\Delta e_{t}b_{t}^{P}$	$\Delta h_{_t}$	$\Delta e_t s_t$	$r_{t-l}b_{t-l}^P$	$r_{t-1}e_{t-1}b_{t-1}^F$
g_{t}	100							
t_{t}	55	100						
Δb_{t}^{P}	45	16	100					
$\Delta e_t b_t^P$	36	37	38	100				
$\Delta h_{_t}$	0	11	7	-10	100			
$\Delta e_t s_t$	-22	2	-20	4	41	100		
$r_{t-1}b_{t-1}^P$	-17	7	10	-6	-5	11	100	
$r_{t-1}e_{t-1}b_{t-1}^F$	-7	18	13	1	-7	15	95	100

The final model, Equation (23), takes one lag of the dependent variable as the regressor, highlighting the short-term impact of current government expenditures on the next period only. Apart from this, the short-term impact is found in tax revenues and the foreign loans after the change in the exchange rate regime. The estimated coefficients are based on the heteroscedasticity and autocorrelation consistent coefficient covariance matrix.

Post-estimation tests for checking the stability of estimated parameters using CUSUM and CUSUM-square tests, as shown in Figure 1 and Figure 2, respectively, confirm that the parameters are stable.

TABLE 4Estimation results of the ARDL Model

Variable	Coefficient	Coefficient Value	Std. Error	t-Statistic
Constant	Constant	0.0494***	0.0173	2.8559
g_{t-1}	Previous Year Government Expenditures	0.6928***	0.0762	9.0914
$t_{t\text{-}I}$	Previous Year Tax Revenue	-0.5525*	0.2752	-2.0073
t_{t}	Current Tax Revenues	0.5482*	0.2925	1.8742
Δb_t^P	Government Bond Hold by Public	0.1700***	0.0540	3.1469
$\Delta e_{_t} b_{_t}^{_P}$	Foreign Bond Holding	-0.0815**	0.0382	-2.1326
$\Delta h_{_t}$	High Power Money	0.2147***	0.0681	3.1526
$\Delta e_t s_t$	Foreign Reserve in Domestic Currency	-0.2692***	0.0660	-4.0797
$\Delta e_{\scriptscriptstyle t-2} b_{\scriptscriptstyle t-2}^{\scriptscriptstyle F} * d_{\scriptscriptstyle t-2}$	Previous two year Real Interest Paid on Foreign Bond	-0.1826**	0.0834	-2.1889
$\Delta e_{\scriptscriptstyle t\text{-}I} b_{\scriptscriptstyle t\text{-}I}^{\scriptscriptstyle F} * d_{\scriptscriptstyle t\text{-}I}$	Previous year Real Interest Paid on Foreign Bond	0.2068***	0.0539	3.8352
$\Delta e_{t}b_{t}^{F*}d$	Current Real Interest Paid on Foreign Bond	0.1589**	0.0637	2.4948
\mathbb{R}^2		0.8497		
Adjusted R ²		0.8013		
F-Statistics		17.5306***		
Durbin Watson		2.1235		

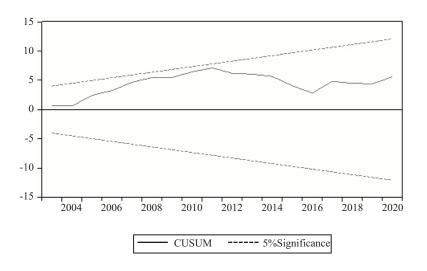


FIGURE 1
Results of CUSUM

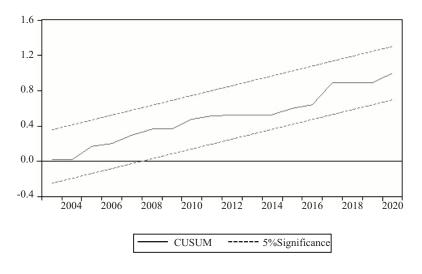


FIGURE 2Results of CUSUM of Squares

The estimated parameters reflect the impact of regressors on dependent variables. We need to keep it clear that the variables used in the model are constructed as percentages of nominal GDP. Therefore, a parameter value shows the proportion of the total impact on a 1 per cent change in government expenditures to nominal GDP due to the regressor. Value of shows that the government expenditures to nominal have been increasing by 5 per cent, on average, over the period, irrespective of any change in the determinants of the spending. Parameters reflect the long-term impact of the determinants on government expenditures and show that the long-term proportion of government expenditures attributed to the financing from tax revenues has a coefficient value of 0.5482, reflecting that nearly 55 per cent of the government expenditures have been financed through tax revenues on average. As indicated, over 21 per cent of government expenditures have been financed by high-powered money. It is followed by a 17 per cent share of financing, as indicated by government bonds held by the public. The foreign bonds/loans are found to have a proportion of -8 per cent, as shown by the fixed exchange rates regime till 1999. However, the proportion reversed after the change in the exchange rate regime from fixed to market-determined and shows an 8 per cent proportion coming from financing government expenditures. As theorised in the model of shrinking government expenditures, the reserves have a share of -27 per cent, as indicated by reserves and foreign bonds/loans are associated with exchange rates. Hence, we estimated both variables with the tax regime dummy. The impact of change in tax regime is not found significant on reserves. The five determinants of government expenditures comply with the a priori information in longterm impact as modelled in this paper.

Apart from the long-term impact, the short-term effect on government expenditures is attributed to three variables: Government expenditures, tax revenues, and foreign bonds/loans. The short-term impact of previous periods' government expenditures shows a consistent positive impact on current government expenditures, but the impact of last period tax revenues and foreign bonds/loans shows an inconsistent pattern.

VI. Conclusions and Recommendations

This paper intends to extend the government expenditures model developed by Walsh (2009) by incorporating the dynamics of an open economy, which finances its government expenditures from foreign loans besides tax revenues and domestic public debt. Thus, it opens room for the impact of foreign reserves and exchange rates due to the addition of foreign loans as a source of financing. The final extended model developed in this paper explains government expenditures as a function of tax revenues, net change in private bond holdings, foreign debt, high-powered money, foreign reserves held by the central bank, and exchange rate. As a priori information, the model suggests that all the variables positively affect government expenditures

except for the change in foreign reserves, interest payments on bonds held by the private sector and interest on foreign debt, which reduces government expenditures. The scope of this paper is limited to modelling government spending by incorporating foreign exchange, foreign reserve and bonds only under the modelling framework as discussed in the initial section of the paper.

Estimating the model for Pakistan as a developing and open economy, on the data from 1976 to 2020, suggests that in the long term, 55 per cent of government expenditures has been financed through tax revenues, followed by 21 per cent by high-powered money, and 17 per cent of the public expenditures financed through the government bonds issued to the private sector, and the foreign bonds loans. Reserves have negatively impacted government expenditures. The exchange rate regime change in 1999 has also impacted government expenditures positively. In the long term, major financing comes from printing money and domestic and foreign debt after tax revenues. An interesting conclusion from the estimation is the higher negative proportion attributed to a change in reserves, which shows that converting foreign reserves, including private and public holdings, into domestic currency almost nullifies the combined effect of financing in terms of foreign loans and high-powered money. We couldn't significantly increase government expenditures to nominal GDP due to foreign loans and money printing.

There is no consistent short-term impact of any determinant on government expenditures. Only previous-period government expenditures are found to affect current-period government expenditures positively. This inconsistency is also apparent from the descriptive analysis of variables. We may argue against active fiscal efforts to finance government expenditures as the fiscal deficit in Pakistan has been covered by one or the other remaining determinants of government expenditures.

Based on the above conclusions, it may be arguably recommended to rationalise the financing of public sector expenditures by focusing more on taxes rather than other sources. The government's effort to tap the private sector's foreign reserves may reduce spending space. It needs to leave the private sector's reserves on the market dynamics.

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