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CONSUMPTION PLANNING:
A suggested New Approach

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This note attempts to show that income elasticities of demand for various consumer goods are affected by the distribution of income. Therefore, any meaningful consumption planning excercise must incorporate the perceived distribution of income over the plan horizon.

I. Introduction

The calculation of income elasticities of demand for various consumer goods is an important part of all consumption planning exercises. Without knowledge of these elasticities it is not possible to determine the future levels of demand and what the levels of investment must be to cater to this demand. Therefore, the accuracy of consumption and investment plans hinges on whether the income elasticities of demand fully take into account the changes in income levels, tastes and other socio-economic factors which are likely to affect consumption patterns over time.

In the case of most developing countries, one of the key factors that influences the growth of consumption demand, particularly for food items, is the nature of change in the distribution of income that is expected to occur during the plan period. It is common knowledge that if the increment in national income goes more in favour of the lower income groups in the population, then this is likely to lead to a faster growth in demand for food items than if the growth in the nationl income is directed towards the upper income groups. In Pakistan the impact of changes in the distribution of income on the growth of demand for consumer goods has not been considered in the consumption plans framed in the various Five Year Plans. Given the fact that the country stands committed toward achieving a more egalitarian order, it becomes even more necessary to allow explicitly for this factor. Therefore, it is felt that the next Five Year Plan of Pakistan, which is in the process of preparation, should incorporate well-defined targets for the increments in income/consumption for different income groups as has been done

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in plans of other countries.[1] These targets will then make it possible to frame a consumption plan which is consistent with the achievement of these targets. Otherwise, there is every likelihood that income elasticities of demand will be used which have been calculated from the data of periods when the distribution of income may have moved in a significantly different manner. Investment plans based on such elasticities will cause the gap between the growth of demand and supply to widen through time. In the case of commodities where the growth of demand is understated, shortage of production capacity will appear and inflationary pressures will be generated and/or lead to greater volume of imports. In the case of items where the growth of demand is overstated, a situation of excess capacity will be created. Altogether, the probability of a maldistribution of investment in the economy will be increased.

This paper presents estimates of income elasticities of demand for twenty-nine consumer goods under alternative assumptions regarding the path that can be followed by the distribution of income. The objective is to highlight how sensitive elasticity estimates are to the nature of changes in the distribution of income.

The paper is organized as follows. Section II gives the theortical rationale behind the method of estimation of the income elasticities of demand. In this section the alternative paths that can be followed by the distribution of income are defined and the actual method of estimation is set out. Section III gives the data sources and the results. Also enclosed is a Mathematical Appendix to the paper. In the appendix are set out in detail the means of arriving at the income elasticities of demand under alternative assumptions regarding changes in the distribution of income.

II. Method of Estimation

Estimates of the income elasticities of demand for different consumer goods have generally been made on a cross-sectional basis by studying the consumption patterns of a sample of households having different levels of income. Very few estimates of income elasticities have been arrived at by studying consumption patterns over time. Such an approach is rendered difficult by econometric problems. Prices change through time, affecting the levels of demand for different commodities. However, these prices are deter-

[1]This was done by the Indian Planning Commission in the formulation of its latest Five Year Plan. The target was to increase the per capita consumption of the lowest 30 per cent of the income recipients by 60 per cent over the five year period.

mined by the interaction between supply and demand. Given this element of simultaneity, therefore, estimation of income elasticities in this manner is fraught with conceptual and econometric problems. In order to abstract from the price effect, the cross-sectional approach is adopted. It is assumed that the same set of prices confront each household when it is making its consumption decisions. Further, when cross-sectional income elasticities of demand are used for forecasting purposes, another basic assumption has to be made. It is that as different income groups move up on the income scale through time, they exactly emulate the consumption pattern of the households which had earlier been on the same income level. In a hierarchical society where the "demonstration effect" on consumption standards is very strong, this assumption may be, more or less, valid in the short run and perhaps also in the medium run. However, over the long run tastes and preferences are bound to change. Therefore, income elasticities estimated from cross-sectional data may be used for forecasting demand only on a short-term and, at best, on a medium-term basis. Long-term forecasts made on the basis of these elasticities are likely to be suspect.

Cross-sectional estimation of the income elasticities of demand is frequently done in the following manner. It is assumed that the income elasticity of demand, ϵi, for each income group in the population is the same, say b. This means that if c_{ij} is the per capita consumption of the jth item in the ith income group with per capita income y_i, then

$$\frac{dc_{ij}/c_{ij}}{dy_i/y_i} = \epsilon_j = b_j \quad i = 1, \ldots, n, \quad j = 1, \ldots, m$$

This transforms to

$$\log c_{ij} = a_j + b_j \log y_i + u_j$$

where u is the random error term. The values of a_j and b_j can then be estimated on the basis of a regression of the logs of the observed values of c_{ij} and y_i for each income group. b_j represents the income elasticity of demand.

However, this method of estimation rests on one major assumption, i.e., that the income elasticities of demand for each income group are the same. In other words, it is assumed that if the income of all house-holds grew at the same rate, then their consumption of any item would also increase at the same rate, irrespective of their income level. Clearly, this is not the case. It can be expected that if the income of the lower income groups was increased at the same rate as that for the upper income groups, the former are likely to spend a considerably greater part of their incremental

income on the consumption of basic necessities like food and clothing while the latter would devote a greater part of their additional income to the consumption of luxury items. It is this fundamental difference in the consumption response to an increase in income of different income groups which makes it essential to incorporate consideration of changes in the distribution of income into the calculation of the overall income elasticities of demand. It is, therefore, postulated that

$$\epsilon_{ij} = f_j (y_i)$$

i.e., that the income elasticity of demand of the ith income group is a function of its income level. The assumption is made that the relationship is either linear or quadratic. More complex formulations could, of course, be tried. In the linear case

$$\epsilon_{ij} = a_j + b_j y_i$$

Depending upon whether b_j is negative or positive, the income elasticity of demand will decrease or increase as the income level increases. *A priori,* it may be expected that in the case of most food items, b_j would be negative. Conversely, for luxury items it is likely that b_j is positive. Further, this formulation implies that for items where b_j is negative there exists a certain income level, y*, beyond which consumption of the item declines as the income goes up. This income level is given by $-a_j/b_j$. In other words, beyond y*, the item becomes an "inferior good".

In the quadratic case

$$\epsilon_{ij} = a_j + b_j y_i + c_j y_i^2$$

There are four alternative possibilities here:

(i) $b_j < 0$, $c_j < 0$: In this case, the income elasticity of demand declines as the income level increases. The rate of decline is greater the higher the income level.

(ii) $b_j < 0$, $c_j > 0$: In this case the income elasticity of demand declines up to a certain income level and then starts increasing.

(iii) $b_j > 0$, $c_j < 0$: In this case the income elasticity of demand rises up to a certain income level and then starts falling.

(iv) $b_j > 0$, $c_j > 0$: In this case the income elasticity of demand rises as the income level increases. The rate of increase is greater the higher the income level.

Of particular interest are cases (ii) and (iii).

The linear formulation of the income elasticity function implies that

$$\frac{dc_{ij}/c_{ij}}{dy_i/y_i} = a_j + b_j y_i + v_j$$

which transforms to

$$\log c_{ij} = k_j + \log y_i + b_j y_i + u_j$$

Similarly, the quadratic relationship transforms to

$$\log c_{ij} = k_j + a_j \log y_i + b_j y_i + \tfrac{1}{2} \cdot c_j y_i^2 + u_j$$

The values of the coefficients in the equations can be estimated by multiple regression. Both regressions have been run for each commodity to see whether a constant, linear or quardratic relationship exists between the income elasticity of demand and the income level.

Following the derivation of the income elasticity function, the next stage in the estimation process is to specify what the growth rates of income are for households with different income levels. This can only be done when the dynamics of the distribution of income in the future is specified. There are clearly three paths that could be followed by the distribution of income through time. It could become more equitable, remain unchanged, or become less equitable. The first path is referred to as the "egalitarian" path; the second as the "neutral" path; and, the third as the "inegalitarian" path. Each path, however, has to be defined in such a manner that the growth rate, g_i, of income for each income group can be specified. This is assumed to be related to income in the following manner:

For the "egalitarian" path: $g_i = k/y_i^{\alpha}$, i.e., g_i is higher the smaller y_i is;
For the "neutral" path: $g_i = k$, i.e., g_i is the same for all income groups;
For the "inegalitarian" path: $g_i = ky_i^{\alpha}$, i.e., g_i is higher the greater y_i is;

where k and α are positive constants and y_i the base year income levels. In this type of specification, it is also assumed that existing income is not redistributed. In other words, changes in the distribution of income are brought about only via the additional income generated in the economy. The value of the constant k depends upon the overall rate of growth of national income. The higher it is the greater is the value of k. The magnitude of α, on the other hand, depends upon how rapidly it is proposed to change the distribution of income. It is beyond the scope of this paper to indicate the

nature of policies to bring about changes in the distribution of income. Further, it does not examine the impact on consumption patterns of policies which improve the living standards not directly by raising income levels but indirectly, e.g., through the provision of electricity in villages. The availability of electricity is clearly likely to reduce the demand for kerosene oil in rural areas.

It has been recommended in the introduction that incremental income targets be specified for different income groups in the future medium-term and long-term plans. If these targets are, in fact, made explicit, then it will become possible to arrive, by a process of *iteration,* at that value of α which generates values of g_is that approximate as closely as possible to the income-growth targets.

The mathematical property of the "egalitarian" path is that in the long run it converges to an egalitarian distribution of income, i.e., all households[2] have roughly the same income level. It is flexible in that the value of α can be changed in accordance with the desired pace of income redistribution. Similarly, the mathematical property of the "inegalitarian" path is that it converges to a state where the bulk of the income in the economy is pre-empted by the uppermost income group. Here again, the specification is flexible enough to allow for changes in the rate of growth of income inequality over time.

Given knowledge of the income elasticities of demand, the rates of growth of income at different income levels, and alternative paths of the distribution of income, it becomes possible to calculate the overall income elasticities of demand. The primary object of this paper is to highlight elasticities of demand for each commodity for the three alternative distribution-of-income paths. Details of the estimation of the elasticities are set out in the Mathematical Appendix.

III. Data Sources and Results

The primary source of data on consumption patterns was the *Household Income and Expenditure Survey* carried out annually since 1966-67 by the Central Statistics Division of the Government of Pakistan. The latest published survey results are for 1971-72. Households are bracketed into thirteen income groups. A "pooled time series-cross-sectional" approach has been adopted in order to have enough degrees of freedom in the regression analysis. Consumption patterns for the period 1969-70 to 1971-72 have been analyzed. An overall consumer price index has been constructed for Pakistan since 1969-70 by the Central Statistics Division. This index has been used for deflating incomes while expenditure figures for different items have been deflated by the respective retail price index. Since the period analyzed

[2]Standardized for the number of members in the household.

is only two years and it happens to be a period when inflation had not fully set into motion in Pakistan and relative prices are unlikely to have changed significantly, this bunching of consumption patterns revealed by three consecutive surveys for study should not introduce the econometric problems mentioned earlier. On the other hand, it provides for three times as many observations.

The values of the coefficients as obtained by multiple regression analysis are shown in Table 1. As shown in the Table, in sixteen out of twenty-nine cases the income elasticity of demand appears to vary significantly with the income level. In eleven cases the relationship appears to be linear while in five it is quadratic. Out of the thirteen food items that were studied, the income elasticity appears to be significantly related to the income level for seven items. These seven items are wheat, rice, pulses (other than gram), milk, ghee (desi), mutton and onions. In the case of industrial consumer goods, the principal items for which the income elasticity of demand appears to vary significantly are sugar (refined), cigarettes, tea, cloth and matches. Therefore, there does appear to be a marked tendency for the income elasticity of demand for some consumer goods, particularly those which weigh heavily in consumption patterns, to be dependent upon the income level.

The results indicate that there are four items in which the income elasticity of demand initially declines with the income level and then starts rising. These items are rice, sugar (refined), cigarettes and cloth. In the case of onions only the income elasticity initially rises and then falls with the rise in income level.

In an earlier section it had been stated that the specification of the income elasticity function implies that certain items can become "inferior goods" beyond a certain income level, i.e., that their consumption declines as income increases beyond that level. Our results indicate that there are two such items. The first item is wheat where the income level, y^*, ranges from Rs. 750 to Rs. 1000 p.m. in 1971-72. The other item is desi ghee where y^* ranges from Rs. 1500 to Rs. 2000 p.m.

An attempt has also been made to construct income classwise income elasticities of demand. This has been done to highlight the differences in the consumption response of different classes in the population to a change in their income levels. Four classes have been identified. Class I has been taken to include households with a monthly income of less than Rs. 300. Class II includes households with monthly income levels ranging from Rs. 300 to Rs. 750. Class III consists of households with monthly income levels of Rs. 750 to Rs. 1500 and, finally, Class IV, with monthly income levels above Rs. 1500. These categories are, of course, arbitrary. However, they are useful in demonstrating the significant differences in income elasticities of demand among different income classes.

These elasticities are shown in Table 2. They have been estimated for the sixteen consumer goods for which the income elasticity appears to

TABLE 1

Results of best regression equation*:

Either Linear, i.e., $\log_e C = k + a \log_e y + by + u$
Or Quadratic, i.e., $\log_e C = k + a \log_e y + by + \frac{1}{2} \cdot cy^2 + u$

	Nature of Relationship	k	a	b	½·c	$\bar{R}^2$	Degrees of Freedom (D. of F.)
1. Wheat	Linear	2.20	0.12	-0.32×10^{2}	—	0.55	36
		(14.31)	(2.50)	(−5.13)			
2. Rice	Quadratic	−4.30	1.56	-0.28×10^{-1}	0.52×10^{-4}	0.66	35
		(5.89)	(5.58)	(−3.68)	(3.11)		
3. Other Pulses	Linear	−2.03	0.29	-0.14×10^{-2}	—	0.75	35
		(−15.47)	(6.55)	(2.23)			
4. Milk	Linear	−2.43	1.35	−0.02	—	0.50	36
		(−3.69)	(6.30)	(−6.11)			
5. Desi Ghee	Linear	−0.29	0.06	-0.65×10^{-2}	—	0.72	36
		(−0.90)	(0.54)	(−4.89)			
6. Mutton	Linear	−1.72	0.38	0.13×10^{-2}	—	0.96	36
		(−15.70)	(10.60)	(3.01)			
7. Onions	Quadratic	0.22	−0.13	0.63×10^{-2}	$-.94 \times 10^{-5}$	0.86	35
		(1.38)	(−2.04)	(3.83)	(−2.55)		

8. Sugar (Refined)	Quadratic	−6.06 (−16.49)	1.67 (11.84)	-0.15×10^{-1} (−3.93)	0.17×10^{-4} (2.05)	0.96	35
9. Cigarettes	Quadratic	−9.36 (−13.70)	2.74 (10.49)	-0.33×10^{-1} (−4.72)	0.59×10^{-4} (3.77)	0.95	35
10. Tea	Linear	−5.44 (−32.85)	2.74 (16.16)	-0.33×10^{-1} (−2.74)	—	0.95	35
11. Cloth	Quadratic	0.67 (2.67)	-0.49×10^{-1} (−0.51)	0.17×10^{-1} (6.63)	0.33×10^{-4} (5.87)	0.97	35
12. Readymade Garments	Linear	−2.43 (−8.62)	0.51 (5.42)	0.28×10^{-2} (2.21)	—	0.88	34
13. Household Textiles	Linear	−4.04 (−17.19)	0.93 (12.47)	-0.28×10^{-2} (−3.30)	—	0.94	34
14. Kerosene Oil	Linear	−2.81 (−16.21)	0.49 (8.73)	-0.23×10^{-2} (−3.19)	—	0.82	35
15. Matches	Linear	−2.63 (−19.37)	0.02 (0.55)	-0.26×10^{-2} (−4.67)	—	0.78	34
16. Furniture & Fixtures	Linear	−5.71 (−10.82)	0.88 (5.04)	0.62×10^{-2} (2.64)	—	0.88	34

Note: t-ratios are enclosed in brackets.

*There is no significant linear or guadratic relationship for a number of items, i.e., that the income elasticity appears to remain constant for all levels of income. These items and their constant income elasticities (shown in brackets) are as follows: gram (−0.01), vegetable ghee (0.60), beef (0.30), fish (1.24), chiken (1.02), eggs (1.68), potatoes (0.34), salt (−0.10), footwear (0.55), kitchen equipment (1.77), crockery & cutlery (0.76), household effects (1.25), personal effects (1.57).

vary significantly with the income level. There appear to be considerable differences in the magnitude of these elasticities. In the case of milk, for example, the income elasticity of demand of Class I is one but approaches zero for Class IV. On the other hand, for furniture and fixtures, the elasticity for Class I is 1.02 which doubles to 2.06 for Class IV.

TABLE 2

Income classwise income elasticities of demand for each consumer good

Consumer Good	Class I 0-300 Rs. p.m.	Class II 300-750 Rs. p.m.	Class III 750-1500 Rs. p.m.	Class IV 1500 Rs. p.m. & above
1. Wheat	0.05	0.01	–0.10	–0.41
2. Rice	0.95	0.70	0.14	0.00
3. Other Pulses	0.25	0.24	0.19	0.05
4. Milk	0.96	0.77	0.27	0.00
5. Desi Ghee	0.43	0.23	0.12	–0.58
6. Mutton	0.42	0.43	0.48	0.62
7. Onions	0.02	0.13	0.22	0.39
8. Sugar (Refined)	1.32	1.17	0.79	0.15
9. Cigarettes	1.99	1.68	0.99	0.67
10. Tea	0.83	0.80	0.74	0.54
11. Cloth	0.32	0.48	0.62	0.78
12. Readymade Garments	0.57	0.61	0.71	1.00
13. Household Textiles	0.86	0.83	0.74	0.42
14. Kerosene Oil	0.44	0.41	0.40	0.17
15. Matches	0.06	0.09	0.09	0.38
16. Furniture & Fixtures	1.03	1.11	1.12	2.06

Finally, in Table 3 are shown the magnitude of the elasticities given different assumptions regarding movements in the distribution of income. The value of the parameter α has been taken as one. This has two advantages. Firstly, it simplifies somewhat the estimating equations shown in the Mathematical Appendix. Secondly, it has a meaningful economic interpretation. Taking the value of α as 1 implies that in the case of the egalitarian path at each time period the *absolute* increase in income per capita is the same

irrespective of the income level.[3] In other words, the egalitarian objective is completely met with regard to the distribution of the additional income generated in the economy. It ensures that over time the income levels of different income groups converge towards one another. If it is felt that taking $\alpha = 1$ represents an unrealistically rapid movement towards an egalitarian order, then it needs to be emphasized that the income growth specification provides only for the redistribution of the *additional* income generated in the economy. Existing levels of income are left untouched. More radical policies designed to achieve greater social justice would almost certainly seek to change the existing pattern of ownership of the means of production by nationalization, land reform, etc.

TABLE 3

Overall income elasticities of demand for each consumer good for different paths followed by the distribution of income

Consumer Good	Egalitarian Income Elasticity of Demand (ϵ_E)	Neutral Income Elasticity of Demand (ϵ_N)	Inegalitarian Income Elasticity of Demand (ϵ_I)
1. Wheat	0.04	0.02	0.01
2. Rice	0.99	0.78	0.44
3. Other Pulses (Ex: gram)	0.23	0.19	0.12
4. Milk	0.99	0.82	0.49
5. Desi Ghee	0.43	0.32	0.11
6. Mutton	0.34	0.46	0.68
7. Onions	0.05	0.07	0.10
8. Sugar (Refined)	1.24	1.12	0.78
9. Cigarettes	1.66	1.58	1.29
10. Tea	0.84	0.79	0.75
11. Cloth	0.42	0.46	0.50
12. Readymade Garments	0.62	0.63	0.66
13. Household Textiles	0.90	0.81	0.61
14. Kerosene Oil	0.47	0.41	0.27
15. Matches	0.07	0.08	0.25
16. Furniture & Fixtures	0.98	1.32	0.47

[3] In the egalitarian path, $g_i = k/y_i^{\alpha}$. In the special case when $\alpha = 1$, $g_i = k/y_i$. This implies that $g_i y_i = k$. Therefore, the increase in income for each income group is the same and is equal to k.

Table 3 demonstrates the degree of sensitvity of the income elasticity estimates to alternate distribution-of-income paths. In most cases the difference between the egalitarian and the neutral elasticity is in the range of ± 15 percent. The difference between the inegalitarian and the neutral elasticities lies generally between ±50 percent. As expected, the maximum difference is between the egalitarian and the inegalitarian elasticities of demand. In some cases, e.g., rice and milk, the former is more than double the latter. In other cases, e.g., furniture and fixtures, the egalitarian elasticity is about two-thirds of inegalitarian elasticity. Altogether, the difference between the three elasticities is sufficiently large for many items to warrant explicit consideration of changes in the distribution of income in projections of demand.[4]

Since our elasticities have been calculated with 1971-72 as the base year, it is possible to compare the magnitude of the estimated egalitarian income elasticitis with the "implicit" income elasticities for the major consumption items in the Fourh Plan. For cereals, it appears that the growth of demand for wheat was overstated, while that for rice was understated. In the case of livestock products, the implicit elasticities are generally higher. For manufactured consumer items, the egalitarian elasticities are higher in the case of sugar (refined) and tea and lower in the case of cloth.[5]

IV. Conclusions

This paper has tried to highlight the importance of incorporating changes in the distribution of income in the projections of demand for consumer goods. It has been demonstrated that in the case of many of the principal consumer items, particularly of food, the magnitude of the income elastici-

[4]If, for example, it is assumed that the target growth of real per capita income in the plan period is 4% per annum while population grows at the rate of 3½% per annum, then if the distribution of income moves along the egalitarian path, the growth of demand over the plan period for, say, milk will be 43.3%. However, if the inegalitarian path is followed, the growth of demand will be 30.4%. The difference between the growth of demand in the two paths is as much as 12.9%.

[5]The implicit elasticities in the Fourth Plan for West Pakistan and our egalitarian elasticities for some of the major consumption items are as follows:

	Implicit Income Elasticity	Egalitarian Elasticity
Wheat	0.61	0.04
Rice	0.61	0.99
Milk	1.10	0.99
Sugar (Refined)	0.79	1.24
Tea	0.75	0.84
Cotton Cloth	1.10	0.42

ties of demand is highly sensitive to assumptions regarding the movement of the distribution of income in the future. Given the national commitment to the achievement of a more egalitarian order, it is recommended that explicit targets be formulated in future plans for growth in income and/or consumption for different income groups in the population. These targets can then form the basis for consumption planning. This will ensure not only that the consumption plans evolved are consistent with the realization of these targets but also that the probability of maldistribution of ivnestment in the economy is minimized.

Mathematical Appendix

In this appendix are set out the means of arriving at the equations for estimating the income elasticity of demand, given that alternative paths are followed by the distribution of the income:

Let us suppose in the base year that

n_i	=	numbe. of individuals in the ith income group	$i = 1, \ldots, n.$
y_i	=	per capita real income in the ith income group	$i = 1, \ldots, n.$
c_{ij}	=	per capita real consumption of the jth commodity in the ith income group	$i = 1, \ldots, n.$ $j = 1, \ldots, m.$

To start off, it is assumed that the income elasticity of demand, ϵ_{ij}, of the ith income group for the jth commodity is given by

$$\begin{aligned} &\text{either,} \quad \epsilon_{ij} = a_j + b_j y_i \\ &\text{or} \quad \epsilon_{ij} = a_j + b_j y_i + c_j y_i^2 \end{aligned} \tag{1}$$

It is then assumed that additional income generated in the economy can be distributed in one of three ways. The first type of path that can be followed by the distribution of income is referred to as the "egalitarian path". According to this path, the growth rate, g_i, of the per capita income of the ith income group is higher the lower the per capita income is in the base

year. Therefore, in this case

$$g_i = \frac{k}{y_i^{\alpha}} \tag{2}$$

where α and k are positive constants.

The second path is referred to as the "neutral path". According to this path, the growth rates of per capita income for all income groups are the same, i.e.

$$g_i = k \tag{3}$$

The third path is called the "inegalitarian path". According to this path, the growth rate, g_i, of per capita income of the ith income group is lower the lower the per capita income is in the base year. Therefore, in this case

$$g_i = ky_i^{\alpha} \tag{4}$$

1. "Egalitarian Elasticity":

Given that the distribution of income follows the path shown in (2), the overall increase in income, ΔY_i, for the ith income group is given by

$$\Delta Y_i = n_i y_i g_i = n_i y_i \frac{k}{y_i^{\alpha}} = k n_i y_i^{1-\alpha} \tag{5}$$

If it is assumed that the overall growth of income for the base year sample of households is g, then

$$\sum_{i=1}^{i=n} \Delta Y_i = g \sum_{i=1}^{i=n} n_i y_i$$

Substituting the value of ΔY_i from (5)

$$k \sum_{i=1}^{i=n} n_i y_i^{1-\alpha} = g \sum_{i=1}^{i=n} n_i y_i$$

This gives the value of k as

$$k = g \frac{\sum_{i=1}^{i=n} n_i y_i}{\sum_{i=1}^{i=n} n_i y_i^{1-\alpha}} \tag{6}$$

Suppose A is defined as

$$A = \frac{\sum_{i=1}^{i=n} n_i y_i}{\sum_{i=1}^{i=n} n_i y_i^{1-\alpha}} \tag{7}$$

Substituting the value of k obtained in equation (6) in equation (2), the value of g_i is

$$g_i = \frac{gA}{y_i^{\alpha}} \tag{8}$$

Since the growth rate of demand, g_{ij}^c, for the jth commodity of the ith income group is given by

$$g_{ij}^c = \epsilon_{ij} g_i$$

Substituting the value of g_i from (8), then

$$g_{ij}^c = \frac{gA\epsilon_{ij}}{y_i^{\alpha}}$$

Therefore, the increase in demand, c_{ij}, of the ith income group is

$$\Delta c_{ij} = \frac{gAn_i\epsilon_{ij}c_{ij}}{y_i^{\alpha}}$$

and the overall increase in demand by the sample will be

$$\sum_{i=1}^{i=n} \Delta c_{ij} = gA \sum_{i=1}^{i=n} \frac{\epsilon_{ij} n_i c_{ij}}{y_i^{\alpha}}$$

implying the overall growth rate of demand, g_j^c, is

$$g_j^c = gA \frac{\sum_{i=1}^{i=n} \frac{\dot{\epsilon}_{ij} n_i c_{ij}}{y_i^{\alpha}}}{\sum_{i=1}^{i=n} n_i c_{ij}}$$

If the egalitarian income elasticity of demand is represented by ϵ^E, then

$$\epsilon_j^E = \frac{g_j^c}{g}$$

i.e.

$$\epsilon_j^E = A \frac{\sum_{i=1}^{i=n} \frac{\epsilon_{ij} n_i c_{ij}}{y_i^{\alpha}}}{\sum_{i=1}^{i=n} n_i c_{ij}} \qquad (9)$$

Finally, in equation (9) can be substituted the value of A given in (7) and the value of ϵ_{ij} given in (1).

2. *"Neutral Elasticity":*

Given that in this case the distribution of income follows the path shown in (3), the overall increase in income ΔY_i, for the ith income group is given by

$$\Delta Y_i = n_i y_i k \qquad (10)$$

As in the egalitarian case, the overall growth of income of the sample is taken as g, implying that

$$\sum_{i=1}^{i=n} \Delta Y_i = g \sum_{i=1}^{i=n} n_i y_i \qquad (11)$$

Equations (10) and (11) imply that;

$$k = g$$

Substituting this value of k into equation (3), then

$$g_i = g \qquad (12)$$

This means that in this case

$$g_{ij}^{c} = g\epsilon_{ij}$$

and the increase in demand, Δc_{ij}, of the income group is

$$\Delta c_{ij} = g\epsilon_{ij} n_i c_{ij}$$

and the overall increase in demand is

$$\sum_{i=1}^{i=n} \Delta c_{ij} = g \sum_{i=1}^{i=n} \epsilon_{ij} n_i c_{ij}$$

implying that the overall growth rate of demand, g_j^c, is

$$g_j^c = \frac{\sum_{i=1}^{i=n} \epsilon_{ij} n_i c_{ij}}{\sum_{i=1}^{i=n} n_i c_{ij}}$$

If the neutral income elasticity of demand is represented by ϵ^N, then

$$\epsilon_j^N = \frac{g_j^c}{g}$$

i.e.

$$\epsilon_j^N = \frac{\sum_{i=1}^{i=n} \epsilon_{ij} n_i c_{ij}}{\sum_{i=1}^{i=n} n_i c_{ij}} \qquad (13)$$

Finally, in equation (13) can be substituted the value of ϵ_{ij} given in (1).

3. *"Inegalitarian Elasticity":*

Given that in this case the distribution of income follows the path shown in (4), then

$$\Delta Y_i = n_i y_i k y_i^{\alpha} = k n_i y_i^{1+\alpha} \qquad (14)$$

and

$$k \sum_{i=1}^{i=n} n_i y_i^{1+\alpha} = g \sum_{i=1}^{i=n} n_i y_i$$

This gives the value of k as

$$k = g \frac{\sum_{i=1}^{i=n} n_i y_i}{\sum_{i=1}^{i=n} n_i y_i^{1+\alpha}} \tag{15}$$

B is defined as

$$B = \frac{\sum_{i=1}^{i=n} n_i y_i}{\sum_{i=1}^{i=n} n_i y_i^{1+\alpha}} \tag{16}$$

Substituting the value of k obatined in equation (15) in equation (4), the value of g_i is obtained as

$$g_i = gBy_i^{\alpha} \tag{17}$$

and, since

$$g_{ij} = \epsilon_{ij} g_i$$

$$g_{ij}^{c} = gB\epsilon_{ij} y_i^{\alpha}$$

Therefore,

$$\Delta c_{ij} = gB\epsilon_{ij} y_i^{\alpha} n_i c_{ij}$$

and

$$\sum_{i=1}^{i=n} \Delta c_{ij} = gB \sum_{i=1}^{i=n} \epsilon_{ij} y_i^{\alpha} n_i c_{ij}$$

implying that

$$g_j^c = gB \frac{\sum_{i=1}^{i=n} \epsilon_{ij} y_i^{\alpha} n_i c_{ij}}{\sum_{i=1}^{i=n} n_{ij} c_{ij}}$$

If the "inegalitarian" elasticity of demand in represented by ϵ^I, then

$$\epsilon_j^I = \frac{g_j^c}{g}$$

i.e.

$$\epsilon_j^I = B \frac{\sum_{i=1}^{i=n} \epsilon_{ij} y_i^{\alpha} n_i c_{ij}}{\sum_{i=1}^{i=n} n_i c_{ij}} \qquad (18)$$

Finally, the value of B as given in (16) and ϵ_{ij} given in (1) are substituted into (18).

In the text, $\alpha = 1$.

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