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# MILK PRODUCTION FUNCTION:
## An Empirical Note*

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Using primary data collected from a survey of cattle farms in Landhi, near Karachi, this note presents results on estimation of production functions for milk for small and large farms. The marginal products for inputs on small and large farms are also presented.

## I. Introduction

Several studies which estimate production and cost functions for dairy industry exist in the case of developed economies [see: Antill (1955), Jarrett (1959), and Wragg and Godsell (1956)]. However, in developing countries similar studies are scant due to dearth of relevant published statistics and unorganized structure of the industry. Except for a few studies conducted on the data obtained from experimental farms, there is almost no literature on the economics of milk production in Pakistan. The present study is based on primary data collected from a survey of cattle farms located in Landhi, near Karachi. It attempts to explain factors responsible for variations in milk production among farms of different size.

## II. Sampling and Survey

Due to heterogeneity in the size of farms (in terms of number of animals)[1] a stratified sample was preferred over a simple random one. Register of number of animals in each farm, maintained by the veterinary hospital constituted the sampling frame. Total number of dairies were divi-

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[1] The number of animals per farm ranged from 6 to 208.

ded into six mutually exculsive strata according to the herd size[2] and the sample was selected from each stratum by the proportional allocation method. Out of a total of 486 dairy units, a sample of 114 units has been used in this study.

Production functions are estimated for the entire sample of 114 units and also for two sub-samples comprising large and small size farms. A unit consisting of 60 or more productive animals[3] is grouped under the category of large farms, while farms with less than 60 animals are termed as small. This division contains more of a subjective element than any theoretical considerations.

A comprehensive questionnaire was administered to each unit by personal interviews. Information was gathered on the number of milch buffaloes, milch cows, input of family and hired labour in terms of hours, types of fodder and concentrates.[4] Being a memory based exercise, the information supplied by respondents was cross-checked for certain key variables. Mostly, the evidence thus obtained corroborated the farmer's information on the corresponding variables.

## III. Production Function

To explain the variation in milk output per day, the following relationship between inputs and output is hypothesized:

$$Q_i = f[B_i, C_i, FL_i, HL_i, DF_i, FG_i, CON_i,], \quad (1)$$

where the subscript i refers to observations on the ith farm, Q is the milk output in Mds (1 Md = 40 kilograms) per day, B is the number of milch buffaloes, C is the number of milch cows, FL and HL are the number of hours of family and hired labour respectively, DF and GF are the amount of dry and green fodder respectively, and CON is an index of different feed concentrates.

The correlation matrix in Table 1 shows the extent and nature of relationships existing among different inputs and between inputs and output. Few observations are in order. Family labour hours exhibit a negative correlation with milk output, which reflects the fact that percentage of family labour hours in total labour hours declines with farm size. One explanation for relatively low correlation between output and milch cows (0.269) may be the smaller proportion of cows in the total milch animals within large

[2] The six strata consisted of the following sizes: 1–20, 21–40, 41–60, 61–80, 81–100, 101 and over.

[3] These include milk animals, dry animals, and animals in their gestation period.

[4] Detailed description of the variables is contained in Discussion Paper No. 42 published by Applied Economics Research Centre, University of Karachi.

**TABLE 1**

Correlation Matrix

| | M. Buff $X_1$ | M. Cows $X_2$ | F. Labour $X_3$ | H. Labour $X_4$ | Dry $X_5$ | Green $X_6$ | Concen-trate $X_7$ |
|---|---|---|---|---|---|---|---|
| Output Y | 0.905 | 0.269 | –0.043 | 0.787 | 0.724 | 0.632 | 0.693 |
| M. Buff $X_1$ | | 0.087 | –0.060 | 0.778 | 0.745 | 0.598 | 0.710 |
| M. Cows $X_2$ | | | –0.016 | 0.212 | 0.137 | 0.134 | 0.156 |
| F. Labour $X_3$ | | | | –0.173 | 0.056 | –0.087 | –0.007 |
| H. Labour $X_4$ | | | | | 0.602 | 0.498 | 0.603 |
| Dry $X_5$ | | | | | | 0.572 | 0.527 |
| Green $X_6$ | | | | | | | 0.452 |

farms. Positive correlation among feed inputs suggests absence of any substitution, though the small coefficients do not support a perfect complementarity. An inverse relationship between family and hired labour is also apparent.

Log-linear (Cobb-Douglas) production function which has been widely used in studies of milk production was selected as the appropriate[5] functional form for (1). Preliminary estimation of (1) by OLS technique led to two modifications in final estimation, the results of which are reported in Table 2. Firstly, milch buffaloes turned out to be more significant than milch cows. This could partly be explained, as mentioned earlier, by the dominant share of buffaloes in the herd population (71% of total animals as compared to only 5% for milch cows). For the purpose of final estimation it was therefore decided to combine them into a single input. Secondly, family labour hours did not have significant impact on output and at the same time were less productive.[6] Therefore, it was decided to combine the two labour inputs into a single input, treating two family labour hours as the equivalent of one hour of hired labour.

In symbols, the modified specification was:

$$Q_i = f(A_i, L_i, DF_i, GF_i, CON_i,), \qquad (2)$$

where A and L represent the total number of milch animals (buffaloes and cows) and labour hours (family and hired) respectively, while the remaining variables stand as before. In addition to estimation of equation (2) for the entire sample, three more equations have been reported in Table 2. Equation (2b) includes a dummy variable meant to capture the effect of farm size. Equations (2c) and (2d) are estimated separately for large and small dairies. The addition of the dummy, however, did not change the results[7] (compare 2a and 2b). By far the largest contributor to milk output per day is the number of milch animals. Surprisingly, the estimated elasticity of milch animals for large farms (0.418) is smaller than similar figure obtained for small farms (1.04). One possible explanation is that in large dairies, a higher proportion of the milch animals are at a lower stage in their lactation cycle, or most of the economies of scale have already been captured in large farms. Next in order of importance is the labour-hours variable, which is

[5] Linear form of equation (1) was estimated and compared with Cobb-Douglas estimation. Based on the criteria of minimum residual sum of squares the Cobb-Douglas formulation performed better and was therefore selected.

[6] The estimated marginal product of family labour was found to be 0.02 Mds. of milk for additional family labour hour as compared to 0.04 Mds. of milk for additional hired labour hour.

[7] Mean of input used and output was observed to be similar with respect to small and large farms.

**TABLE 2**

Production Elasticities

| ITEMS | OVERALL | | SMALL FARM SIZE (2c) | LARGE FARM SIZE (2d) |
|---|---|---|---|---|
| | WITHOUT DUMMY (2a) | WITH DUMMY (2b) | | |
| MILCH ANIMALS | 0.837<br>(9.92) | 0.853<br>(9.29) | 1.040<br>(8.52) | 0.418<br>(3.45) |
| LABOUR HOURS | 0.097<br>(1.81) | 0.097<br>(1.80) | 0.054<br>(0.80) | 0.254<br>(2.95) |
| DRY FODDER | −0.008<br>(−0.52) | −0.009<br>(−0.51) | −0.057<br>(−1.74) | 0.019<br>(1.37) |
| GREEN FODDER | 0.005<br>(0.70) | 0.006<br>(0.75) | 0.005<br>(0.46) | 0.031<br>(2.54) |
| CONCENTRATE | 0.056<br>(1.17) | 0.056<br>(1.18) | 0.008<br>(0.12) | 0.115<br>(2.09) |
| DUMMY | | 0.032<br>(0.45) | | |
| CONSTANT | −1.51<br>(−8.83) | −1.58<br>(−6.42) | −1.91<br>(−7.61) | −0.55<br>(−1.42) |
| $R^2$ | 87.7 | 87.6 | 81.2 | 72.3 |
| F-RATIOS | 163.18 | 134.03 | 59.64 | 23.92 |
| SUM OF ELASTICITIES | 0.987<br>(−0.37) | 1.002<br>(0.37) | 1.050<br>(0.79) | 0.837<br>(−1.61) |
| N | 114 | 114 | 69 | 45 |

t – ratios are in parenthesis

significant in three out of the four equations. The poor performance of labour input in small dairies (equation 2c) can be explained by the importance of family labour in the production structure of such units and the construction of the labour-hours variable itself, which accords more weight to hired labour. Among the feed variables (dry fodder, green fodder and concentrate) the relationship between dry fodder and output is not stable. Only in the case of large farms (equation 2d), its marginal product is positive but not statistically significant. In the remaining equations, (2a, 2b, 2c), it appears as a deterrent to increased production, but here again, the coefficient, statistically speaking, may be zero. The fact that concentrates and green fodder variables are highly significant in the large farms, while the elasticity of milch animals is low, may again reflect the practice of giving higher quality feeds only during periods of relatively high productivity. These results are probably due to specification bias resulting from the failure to include a variable reflecting the composition of herd in terms of their lactation stage.

Assuming that milk production is solely characterised by the inputs under consideration, the sum of elasticities of inputs will give us a point estimate of returns to scale. The last row in Table 2 gives us these totals. For equations (2a) and (2d), the sum of elasticities is less than one, while equations (2b) and (2c) show the sum to be greater than one. A test suggested by Rao & Miller (1971) to determine whether they differ significantly from one (i.e. constant returns) was statistically not significant at the 0.05 level. Hence the hypothesis of constant returns to scale cannot be rejected for milk production.

## IV. Marginal Products

Table 3 gives the marginal product of inputs calculated from the estimated production elasticities using the geometric means of inputs and outputs [see: Heady and Dillon (1967)]. These figures for marginal product indicate the expected change in total output with the addition of a unit of a particular input. The impact is being evaluated at average levels of inputs and output. Comparing the marginal product of milch animals in small and large farms, the contribution to output by an additional animal in the small size dairy is more than twice that of the latter. These differences can be attributed to the composition of the herds, physical characteristics such as age, health or breed and managerial set up. The higher marginal product of dry fodder and concentrate in large farms may be due to greater efficiency in use by large farms or possibly due to their underutilization in relation to other inputs.

## V. Summary of Findings

The main findings can be summarised as follows:

(a) In production relationship the number of milch animals appeared to be the determining factor in explaining milk output per day. However, herds in smaller dairies contributed higher output on

TABLE 3

Marginal Products

| VARIABLES | HERD (OVERALL) | | HERD SMALL | | HERD LARGE | |
|---|---|---|---|---|---|---|
| | GEOMETRIC MEANS (MDS) | MARGINAL PRODUCTS (MDS) | GEOMETRIC MEANS (MDS) | MARGINAL PRODUCTS (MDS) | GEOMETRIC MEANS (MDS) | MARGINAL PRODUCTS (MDS) |
| MILCH ANIMALS | 40.72 | 0.151 | 27.31 | 0.189 | 75.14 | 0.075 |
| LABOUR HOURS | 16.58 | 0.043 | 10.61 | 0.025 | 32.89 | 0.105 |
| DRY FODDER | 5.81 | –0.010 | 4.23 | –0.067 | 9.43 | 0.027 |
| GREEN FODDER | 1.65 | 0.024 | 0.61 | 0.037 | 7.64 | 0.055 |
| CONCENTRATE | 12.21 | 0.034 | 7.06 | 0.006 | 23.07 | 0.067 |
| MILK OUTPUT | 7.37 | | 4.95 | | 13.55 | |

per animal basis than in bigger farms.

(b) In large size dairies, the marginal product of labour is nearly four times that of small farms.

(c) Among feed inputs, the impact of dry fodder in increasing output appears to be negligible. This observation leads to serious doubt on the usefulness of traditional practice of feeding large amounts of dry fodder to milch animals. Effect of dry fodder on milk production needs to be accurately quantified in economic and technical terms, in order to educate extension workers in dairy farming.

(d) Constant returns to scale prevail in both large and small farms.

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