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AN EMPIRICAL TEST OF THE RISK AVERSION HYPOTHESIS

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In this paper we develop a framework for the empirical analysis of production theory under uncertainty. Applying duality theory and using a flexible functional form for technology, we provide a model for production decisions under uncertainty that can be easily estimated and used to identify attitudes towards risk. Furthermore, we provide a measure of degree of risk aversion and a direct test for the risk aversion hypothesis.

I. Introduction

The effects of uncertainty on producers' behaviour have been recently the subject of several studies.[1] These studies show that producers will change their behaviour under uncertainty and moreover the nature of the change in their behaviour will depend on their attitudes towards risk.[2]

The most common assumption about individuals' attitudes towards risk has been that individuals are risk averse. Theoretical support for this assumption, which is related to the boundedness of the Von-Neuman-Morgenstern utility function, is suggested by Arrow (1971). Empirical support is given by observed phenomena such as purchase of insurance, portfolio diversification and other risk-sharing contracts. On the other hand, it has been also argued that since many firms are owned by a large number of shareholders whose portfolios are diversified, risk neutrality of the firm may be appropriate.

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[1] See Sandmo (1971), Batra and Ullah (1974), and Hartman (1976).

[2] See references in footnote 1.

While the theoretical discussion on these issues is quite extensive and developed, there are very few empirical applications[3] of the theory of the firm under uncertainty. Many studies in applied production theory appeared recently in which new functional forms and new techniques are being used.[4] None of these studies, however, considers the existence and effects of uncertainty.

In this paper we develop a framework for the empirical analysis of production theory under uncertainty. Applying duality theory and using a flexible functional form for technology, we provide a model for production decisions under uncertainty that can be easily estimated and used to identify attitudes towards risk. Furthermore, we provide a measure of the degree of risk aversion (or preference) and a direct test for the risk aversion hypothesis.

In the empirical part we apply our framework to a study of the U.S. textile industry (1947-1971) and find risk aversion behaviour to be insignificant.

In section II we provide the theoretical framework, in section III we discuss the econometric specification of the model and in section IV we report the empirical results.

II. Theoretical Framework

Consider a firm whose technology is given by the production function $y = F(x)$ where x is an n vector of variable inputs, y is output and F is a continuous, non-decreasing and concave function. The firm is assumed to be competitive in its input markets and faces the variable input price vector w with certainty. The firm is also assumed to be competitive in its output market; the output price P is, however, a random variable and is unknown when production decisions are made. The firm is, therefore, competitive in a probabilistic sense; it does not face a given output price, but it cannot affect its probability distribution.

The firm's objective is to maximize expected utility from profits and its attitude toward risk is described by a Von-Neuman-Morgenstern utility function $U = U(\pi)$ where U is utility, π profits and $U(\pi) > 0$, $U'(\pi) > 0$. The firm's utility function is concave, linear or convex in π depending on whether the firm is risk averse, risk neutral or risk loving.

The firm's problem can, therefore, be written as:

$$\max_{y,\,x} \{E[U(\pi)] : \ \pi = Py - wx, \ y = F(x), \ P \sim g(P)\ x, y \geqslant 0\} \tag{1}$$

[3] For an empirical application of choice under uncertainty, see Parkin (1970) where a discount house portfolio model is developed.

[4] For references, see Fuss and McFadden (1978), and Diewert (1974).

where E is the expectation operator and g(P) is the probability density function of P and w is a vector of factor prices.

Since uncertainty enters the problem through output price uncertainty only, the problem can be rewritten as the following equivalent problem:

$$\max_{y} \{ E[U(\pi)] : \pi = Py - C(w,y), P \sim g(P), y \geqslant 0 \} \tag{2}$$

where
$$C(w,y) \equiv \min_{x} \{ wx : y = F(x), x \geqslant 0 \} \tag{3}$$

is the cost function which is dual to the production function F.[5] Thus, the firm produces efficiently, although output price is uncertain.[6] The input demand functions can be obtained from (3) by Shephard's Lemma [see Diewert (1971)] as:

$$x = \nabla_w C(w,y) \tag{4}$$

where we assume differentiability of C, and ∇_w is the vector of derivatives of C with respect to w.

The optimality condition for output choice is obtained from (2) as:

$$E[U'(\pi)(P - \frac{\partial C}{\partial y})] = 0 \tag{5}$$

which can be equivalently written as:

$$E(P) - \alpha - \frac{\partial C}{\partial y} = 0 \tag{6}$$

where α, defined by:

$$\alpha = -\operatorname{cov}[U'(\pi),P]/E[U'(\pi)], \tag{7}$$

is the "marginal cost" of uncertainty. Since $U'(\pi) > 0$ and $P > 0$, it is clear that $E[U'(\pi)]$, E(P) are strictly positive. Furthermore $\operatorname{cov}[U'(\pi), P]$ and $U''(\pi)$ have the same sign.[7] In other words the covariance is positive, zero or negative depending on whether the firm is risk loving, risk neutral or risk averse respectively.

This along with the optimality condition (6) implies that the firm will produce more, the same amount, or less output than an identical firm facing

[5] See Diewert (1971) for its regularity properties.

[6] This can be easily verified by observing that the first-order conditions corresponding to (1) yield: $\frac{\partial F}{\partial x_i} E[U'(\pi)P] = w_i E[U'(\pi)]$ and thus, $[\partial F/\partial x_i]/[\partial F/\partial x_j] = w_i/w_j$.

[7] See, for example, Sandmo (1971), Batra and Ullah (1974) and Hartman (1976).

the price E(P) with certainty depending on whether it is risk loving, risk neutral or risk averse respectively (assuming the cost function is convex in output). In other words, the marginal cost of uncertainty, given by α, can be positive, zero, or negative depending on whether the firm is risk averse, risk neutral or risk loving. Thus, by estimating α and testing for its sign we could identify the firm's attitudes towards risk.

It should be noted that the sign of α or $\{\mathrm{cov}[U'(\pi),P]\}$ need not be the same globally. In other words, it will depend on the level of profit π, which in turn depends on the values of the various exogenous variables. Thus, for example, the firm may be risk neutral over some range but risk averse over another; α captures *local* attitudes towards risk.

For empirical implementation we have to specify a functional form for the firm's cost function. Given this functional form we can derive the input demand equations given by system (4) and the output optimality condition given by (6). If the expected price were known and if α could be treated as an unknown parameter, we could simply estimate the system of equations given by (4) and (6) and carry out the desired tests about α. The expected price is, however, unobservable and furthermore, α is not a parameter, but is determined by the observable data and the functional forms of U and C. It is necessary, therefore to make specific assumptions about the relationship between expected and actual output prices and about the form of α. We could assume, for example, that $P = E(P) + r$ where r is some random variable and solve for E(P) in terms of P and r. The random variable r is then simply taken as part of the regression disturbance term. This specification is essentially a form of the rational expectations hypothesis.

The specific form of α will depend, of course, on the characteristics of the firm's utility function and the price distribution. It is, however, clear from the definition of α that in general $\alpha=\alpha(w,y;$ other parameters), i.e., it is "locally" determined by the exogenous variables w, output (which itself, at the optimum, is a function of the exogenous variables and parameters) and the various parameters of the functional forms, including, for example, the variance of the price σ^2.

For an actual application we could approximate α as a function of the various variables and use this approximation in the estimating system. For example, if the probability function g(P) is sufficiently concentrated and compact so that expected utility can be accurately approximated as [see Pratt (1964)]: $EU(\pi) = U(\bar{\pi} - \frac{1}{2} R(\bar{\pi}) y^2 \sigma^2)$ where $\bar{\pi}=E(\pi)$, $R(\cdot) = -U''(\cdot)/U'(\cdot)$ is the Arrow-Pratt measure of absolute risk aversion. Then we can get $\alpha=R\sigma^2 y$ for a constant absolute risk aversion case, or $\alpha = \theta\sigma^2 y$ where $\theta=R/(1-\frac{1}{2}R'y^2\sigma^2)$ for the case when R is not constant.[8] Given the func-

[8] See Appelbaum and Lim (1982).

tional form for α and the expression for E(p), we could then estimate the system (4), (6) and investigate the nature of α, i.e., the firm's attitude towards risk.

III. Empirical Implementation

Having discussed the theoretical framework we now apply it to the U.S. textile industry. We choose the industry because it is considered, and in fact was shown[9] to be, competitive. We assume that there are three competitively priced inputs in the production of textile (y); labour x_L, capital x_K and intermediate goods x_M, whose prices are w_L, w_K and w_M respectively.

The price and quantity series for labour, intermediate inputs and output are constructed from data published in various issues of the *Survey of Current Business*.[10]

We assume that the industry's cost function is given by a translog function:

$$\ell n\, C = \alpha_o + \sum_i \alpha_i\, \ell n\, w_i + \frac{1}{2} \Sigma\Sigma\alpha_{ij}\, \ell n\, w_i\, \ell n w_j + \sum_i \beta_{iy}\, \ell n w_i\, \ell n y + \gamma_y\, \ell n y + \frac{1}{2}\gamma_{yy}\, (\ell n y)^2, \qquad i,j = L,K,M \tag{8}$$

Linear homogeneity in prices and symmetry imply the following parameter restrictions:

$$\Sigma\alpha_i = 1,\ \sum_i \alpha_{ij} = 0\ \forall j,\ \sum_j \alpha_{ij} = 0\ \forall i,\ \sum_i \beta_{iy} = 0,\ \alpha_{ij} = \alpha_{ji} \tag{9}$$

Applying Shephard's Lemma to the cost function (8), we get the input cost share equations as:

$$\begin{aligned} S_L &= \alpha_L + \alpha_{LL}\, \ell n w_L + \alpha_{LM}\, \ell n w_M + \alpha_{LK}\, \ell n w_K + \beta_{LY}\, \ell n y \\ S_M &= \alpha_M + \alpha_{LM}\, \ell n w_L + \alpha_{MM}\, \ell n w_M + \alpha_{MK}\, \ell n w_K + \beta_{My}\, \ell n y \\ S_K &= \alpha_K + \alpha_{LK}\, \ell n w_L + \alpha_{MK}\, \ell n w_M + \alpha_{KK}\, \ell n w_K + \beta_{Ky}\, \ell n y \end{aligned} \tag{10}$$

where S_L, S_M and S_K are the cost shares of labour and materials respectively.

Given the cost function we also obtain the optimality condition (6) as:

$$E(P) = (\gamma_y + \beta_{LY}\, \ell n w_L + \beta_{Ky}\, \ell n w_K + \gamma_{yy}\, \ell n y) C/y + \alpha \tag{11}$$

[9] See Appelbaum (1982).

[10] See Appelbaum (1982) for further references and discussion of data construction.

and using P = E(P)+r (i.e., actual price is given by expected price plus some random term), we get the condition:

$$P = (\gamma_y + \beta_{LY}\, \ell n w_L + \beta_{MY}\, \ell n w_M + \gamma_{yy}\, \ell n y + \beta_{KY}\, \ell n y) C/y + \alpha + r. \qquad (12)$$

As was indicated above, α is not a parameter, but some function of the various variables (and parameters). As a first approximation we take α to be a linear function of these variables. Thus, we write (the structural from equation) for α as:

$$\alpha = (a_o + a_L w_L + a_M w_M + a_K w_K + a_y y)\sigma^2 y \qquad (13)$$

For equations (10) and (12) to be consistent with optimizing behaviour we have to impose the restrictions given by (9). It should also be noticed that since cost shares sum to one, only two of the input share equations in (10) are independent. In the estimation we therefore drop one of the input share equations (the material share equation) and identify its parameters using the adding up (or homogeneity) restrictions. Our full model, therefore, consists of the labour and capital share equations in (10) and the price equation (12), with α being defined by (13).[11]

For empirical implementation the model has to be imbedded within a stochastic framework. To do this we assume that equations (10) and (12) are stochastic due to errors in optimization and define the error term in the input share equations at time t as $e_L(t)$, $e_K(t)$ and the error term in the price-marginal cost decision as $V_p(t)$. The disturbance term in equation (12) is therefore given by $e_p(t) = r(t) + V_p(t)$, i.e., it is the sum of the optimization error and the random deviation of price from expected price. We define the column vector of disturbances of time t as $e(t) \equiv [e_L(t), e_K(t)\ e_p(t)]'$ and assume that the vector of disturbances is identically and independently joint normally distributed with mean vector zero and non-singular covariance matrix Ω,

$$E[e(s)\,e(t)'] = \begin{cases} \Omega & t=s \quad \forall\, s,t \\ o & t \neq s \end{cases}$$

where Ω is a 3 x 3 positive definite matrix.

For estimation we use the full information maximum likelihood technique treating the labour and capital cost shares and the output price as endogenous variables.[12] For statistical inference we use the likelihood ratio test.

[11] In the estimation σ^2 is calculated from the sample.

[12] The standard TSP package was used.

IV. Empirical Results

We estimate the model given by the labour share, capital share and price equations with the symmetry and linear homogeneity restrictions (9) imposed. The model has 14 free parameters, the remaining parameters are identified using (9). Given the parameter estimates we calculate the estimated α and report the figures in Table 1. An examination of these figures shows that the estimated α is positive but very low for all sample points, indicating weak risk averse behavior during the sample period.

To test for the significance of risk aversion we have to test whether α is significantly greater than zero. Given that α is not a constant, but a function of the sample observations, it may be locally (at some points), but not necessarily globally, insignificant. Clearly, a sufficient condition for $\alpha = 0$, is that $a_o = a_y = a_L = a_K = a_M = 0$. This, however, is not a necessary condition. Thus, if the global condition is rejected, it is then necessary to carry out local tests at various sample points.

We estimate our model with and without the global restrictions imposed. The χ^2 statistic for the likelihood ratio test is $2(478.483 - 477.889) = 1.188$ so that for the null hypothesis that all the a's are zero cannot be rejected ($\chi^2_{(5).01} = 15.1$).

Thus, we concluded that while α is positive, it is not significantly positive. We, therefore, accept the hypothesis that the attitude towards risk is characterized by risk neutrality during the sample period.

Finally, we have tried the same model under various alternative assumptions and obtained the same results. We estimated the model assuming that the measure of risk aversion, R, is constant, and when α is a nonlinear (second degree) function of y and w's. We also estimated it assuming that α is alternatively a function of y only, or w only and again in all cases results were the same; that is, we accepted risk neutrality. Finally, instead of estimating the price equation as in (12) we estimated an output share equation again reaching similar conclusions.

TABLE 1
Estimated risk aversion measure α

Year	$\hat{\alpha}$	Year	$\hat{\alpha}$
1947	0.0059	1960	0.0095
48	0.0060	61	0.0098
49	0.0057	62	0.0011
1950	0.0077	63	0.0012
51	0.0087	64	0.0014
52	0.0079	65	0.0017
53	0.0083	66	0.0019
54	0.0068	67	0.0020
55	0.0086	68	0.0024
56	0.0087	69	0.0027
57	0.0083	1970	0.0026
58	0.0077	71	0.0030
59	0.0098		

V. Conclusion

In this paper we provide a framework for the empirical analysis of production theory under price uncertainty. We develop a flexible production model that can be easily estimated and provide a measure for the degree of risk aversion. We also suggest a direct test for the risk aversion hypothesis.

In applying our approach to the U.S. textile industry we find that risk aversion behavior was statistically insignificant during the same period.

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